MAT 133Y1Y TERM TEST #1

THURSDAY, JUNE 9, 2011   7:10 - 9:10 PM

FAMILY NAME: ____________________

GIVEN NAMES: ____________________

STUDENT NUMBER: ____________________

TUTORIAL ROOM: ____________________

Aids Allowed: Calculator with empty memory, to be supplied by the student. Absolutely no graphing calculators allowed.

Instructions: This test has 10 multiple choice questions worth 4 marks each and 5 written answer questions worth a total of 60 marks. For each multiple choice question, you may do your rough work in the test booklet, but you must record your answer by circling one of the letters A, B, C, D or E which appear on the front page of the test. A multiple choice question left blank, or having an incorrect answer circled, or having more than one answer circled, will be assigned a mark of 0. For the written answer solutions, present your solutions in the spaces provided. Use the back of the question pages for your rough work.

GRADER'S REPORT	
Multiple Choice	/ 40
Question 11	/ 15
Question 12	/ 12
Question 13	/ 10
Question 14	/ 10
Question 15	/ 13
TOTAL	/100

ANSWERS FOR MULTIPLE CHOICE Circle the correct answer					
1.	**A**	**B**	**C**	**D**	**E**
2.	**A**	**B**	**C**	**D**	**E**
3.	**A**	**B**	**C**	**D**	**E**
4.	**A**	**B**	**C**	**D**	**E**
5.	**A**	**B**	**C**	**D**	**E**
6.	**A**	**B**	**C**	**D**	**E**
7.	**A**	**B**	**C**	**D**	**E**
8.	**A**	**B**	**C**	**D**	**E**
9.	**A**	**B**	**C**	**D**	**E**
10.	**A**	**B**	**C**	**D**	**E**

1. The effective annual interest rate that is equivalent to a nominal rate of 6% compounded semi-annually is closest to:

let r be the effective annual rate

A 6.09 %

B 5.91 %

C 6.13 %

D 6 %

E 6.2 %

$1+r = (1.03)^2$

$r = (1.03)^2 - 1$

$\simeq .0609$

$\therefore r = 6.09\%$

2. What is the present value of $20,000 in 10 years, if interest is 4% compounded quarterly?

A $13,511.28

B $13,433.06

C $29,777.27

D $20,000

E $29,604.89

$PV = 20{,}000\,(1.01)^{-40}$

$\approx \$13433.06$

3. If money doubles in 8 years, then interest must be compounded continuously at an annual rate that is closest to:

A 7.72%

B 6.25%

C 8.66%

D 9.13%

E 9.05%

Let the amount of money be P.

$$2P = Pe^{8r}$$

$$e^{8r} = 2$$

$$8r = \ln 2$$

$$r = \frac{\ln 2}{8} = 0.0866.$$

$$r \approx 8.66\%.$$

4. A 10-year \$175,000 mortgage has payments of \$1942.86 at the end of each month at 6% compounded monthly. The interest portion of the last payment is closest to:

A \$0

B \$9.67

C \$1942.86

D \$1933.19

E \$9.71

O.P. after 2nd last payment

= P.V. of last payment

$= 1942.86(1.005)^{-1}$

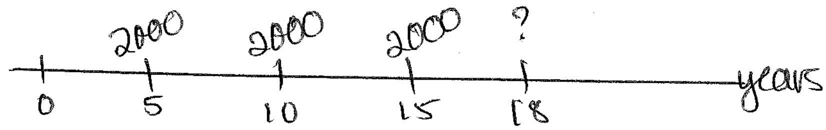
$= \$1933.19$

Interest in last payment $= 1933.19 \times .005$

$= \$9.67$

or Interest $= 1942.86 - 1933.19 = \$9.67.$

5. If a father deposited $2000 on each of his child's 5^{th}, 10^{th} and 15^{th} birthdays into an account earning 3.5% compounded annually, then how much will be in the account on his child's 18^{th} birthday?



A $6,000

B $4601.52

C $7978.97

D $7900.85

E $7714.98

$F.V. = 2000(1.035)^{13} + 2000(1.035)^{8} + 2000(1.035)^{3}$

$\approx \$7978.97$

6. How much money would you have to leave to your kids in order to generate a monthly income of $2000 indefinitely, if interest is 3% compounded monthly? — monthly rate = .0025

A $8,000

B $80,000

C $800,000

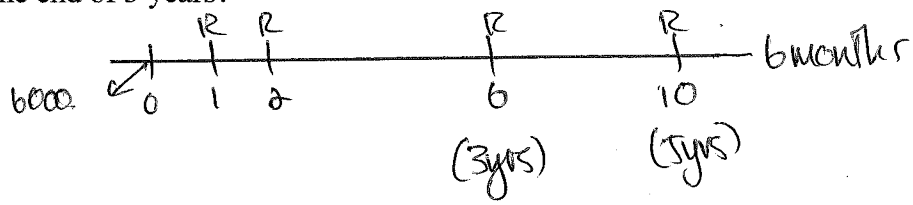
D $200,000

E $66,666.67

$\text{amount} = \dfrac{2000}{.0025}$

$= \$800,000$

7. A \$6000 loan has semi-annual payments for 5 years at 5% compounded semi-annually. How much is still owed at the end of 3 years?



A \$3257.80

B \$2579.02

C \$3600

D \$3776.10

E \$2400

$$R\, a_{\overline{10}|.025} = 6000$$

$$R = \frac{6000}{a_{\overline{10}|.025}} \approx \$685.55$$

O.P. after 6 payments = PV of remaining 4 payments

$$= 685.55\, a_{\overline{4}|.025}$$

$$\approx \$2579.02$$

8. The system :

$$\begin{aligned} x - 2y + 3z &= 2 \\ 2x - 3y + z &= 1 \\ 3x - 4y + kz &= 1 \end{aligned}$$

has no solution if k =

A 19

B −1

C 0

D 1

E 9

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 2 & -3 & 1 & 1 \\ 3 & -4 & k & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 0 & 1 & -5 & -3 \\ 0 & 2 & k-9 & -5 \end{array}\right] \begin{array}{l} R_2 - 2R_1 \\ R_3 - 3R_1 \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 0 & 1 & -5 & -3 \\ 0 & 0 & k+1 & -7 \end{array}\right] R_3 - 2R_2$$

no solution $\Rightarrow k+1 = 0 \Rightarrow k = -1$

9. The following system:

$$\begin{aligned} w + x + y + 5z &= 1 \\ w + 2x + 3y + 8z &= 1 \\ w - x - 3y - z &= 1 \end{aligned}$$

has:

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 8 & 1 \\ 1 & -1 & -3 & -1 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & -2 & -4 & -6 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 0 \end{array}\right] R_2 \& R_3$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 1 \\ 0 & 1 & 2 & 3 & 0 \end{array}\right] R_1 - R_2$$

A only a unique solution

B infinitely many solutions with one parameter

C infinitely many solutions with two parameters

D infinitely many solutions with three parameters

E no solution

$R_1 \Rightarrow w - y + 2z = 1 \Rightarrow w = 1 + y - 2z \quad y \in \mathbb{R}, z \in \mathbb{R}$

$R_2 \Rightarrow x + 2y + 3z = 0 \Rightarrow x = -2y - 3z$ " "

∞ infinite solutions with two parameters

10. If $Ax = C$ where $A^{-1} = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ then $x =$

A $\begin{bmatrix} 6 \\ -7 \end{bmatrix}$

B $\begin{bmatrix} 14 \\ 5 \end{bmatrix}$

C $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

D $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

E $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$$x = A^{-1}C = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

11. A person wishes to purchase a $275,000 mortgage. The going rate is 4% compounded semi-annually and the payments are monthly (starting in one month`s time).

(a) Find the effective monthly rate. Let r = effective monthly rate

$$(1+r)^{12} = (1.02)^2$$

(3) $\Rightarrow r = (1.02)^{1/6} - 1 \approx .00330589$

(b) Find the monthly payment and the total finance charge if the mortgage is for 25 years.

$$R = \frac{275{,}000}{a_{\overline{300}|r}} = 1446.555505 \quad (4)$$

(6) Finance Charge $= 300R - 275{,}000$ (2)

$= \$158966.65$

(or $158968 if you round payment)

(c) Find the monthly payment and the total finance charge if the mortgage is for 15 years.

$$R = \frac{275{,}000}{a_{\overline{180}|r}} = 2029.606455 \quad (4)$$

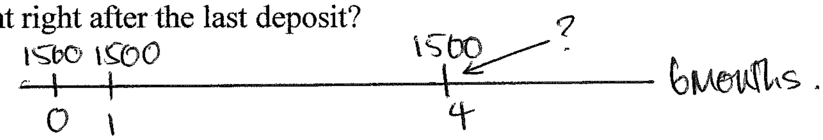
(6)

Finance Charge $= 180R - 275{,}000$ (2)

$= \$90329.16$

(or $90329.80 if you round payment)

12. (a) A person makes 25 semi-annual deposits of \$1500 into an account earning 7% interest compounded semi–annually. If the first deposit is made right away, then how much will there be in the account right after the last deposit?

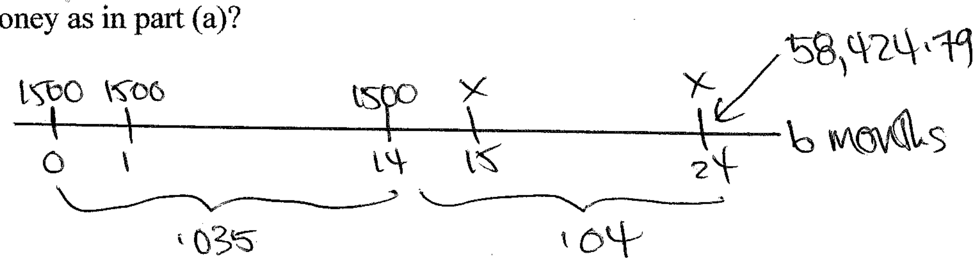


(4)

$$FV = 1500\, s_{\overline{25}|.035} \quad (2)$$

$$\approx \$58{,}424.79 \quad (2)$$

(b) If after 15 deposits have been made, the interest rate changes to 8% compounded semi-annually, then what must each of the last 10 deposits be in order to accumulate the same amount of money as in part (a)?



(8)

$$FV = 1500\, s_{\overline{15}|.035}(1.04)^{10} + X\, s_{\overline{10}|.04} = 58424.79 \quad (4)$$

$$12.00610712\, X \doteq 58424.79 - 42843.48$$

$$X \approx \$1297.78 \quad (4)$$

13. Find the inverse of the following matrix. $A = \begin{bmatrix} 4 & -2 & 3 \\ 8 & -3 & 5 \\ 7 & -2 & 4 \end{bmatrix}$

(10)

$$\left[\begin{array}{ccc|ccc} 4 & -2 & 3 & 1 & 0 & 0 \\ 8 & -3 & 5 & 0 & 1 & 0 \\ 7 & -2 & 4 & 0 & 0 & 1 \end{array}\right] \quad (1)$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 4 & -2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 6 & -5 & -7 & 0 & 4 \end{array}\right] \begin{array}{l} \\ R_2-2R_1 \ (1) \\ 4R_3-7R_1 \ (2) \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 4 & 0 & 1 & -3 & 2 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -6 & 4 \end{array}\right] \begin{array}{l} R_1+2R_2 \ (1) \\ \\ R_3-6R_2 \ (1) \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 4 & 0 & 0 & -8 & 8 & -4 \\ 0 & 1 & 0 & 3 & -5 & 4 \\ 0 & 0 & 1 & 5 & -6 & 4 \end{array}\right] \begin{array}{l} R_1-R_3 \ (1) \\ R_2+R_3 \ (1) \\ \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 2 & -1 \\ 0 & 1 & 0 & 3 & -5 & 4 \\ 0 & 0 & 1 & 5 & -6 & 4 \end{array}\right] R_1 \div 4 \ (1)$$

$$\therefore A^{-1} = \begin{bmatrix} -2 & 2 & -1 \\ 3 & -5 & 4 \\ 5 & -6 & 4 \end{bmatrix}. \quad (1)$$

14. A bank wishes to invest $\$100,000$ in three sources: bonds paying 4% annually, certificates of deposit paying 3.5% annually and first mortgages paying 5% annually. The bank wants to obtain an annual income of $\$4000$ from the three investments. The total amount that the bank invests in bonds and certificates of deposit must be triple the amount invested in mortgages.

(a) If x, y and z represent the amounts invested in bonds, certificates and first mortgages respectively, write the above as a system of equations.

$$x+y+z = 100{,}000 \quad (1)$$
$$.04x + .035y + .05z = 4000 \quad (1)$$
$$x+y = 3z \quad (1)$$

(3)

(b) Solve the above system and hence determine how much the bank should invest in each source.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 100{,}000 \\ 8 & 7 & 10 & 800{,}000 \\ 1 & 1 & -3 & 0 \end{array}\right] \text{Eq } 2 \times 200 \quad (2)$$

(7)

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 100{,}000 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & -4 & -100{,}000 \end{array}\right] \begin{array}{l} \\ R_2 - 8R_1 \quad (1) \\ R_3 - R_1 \quad (1) \end{array}$$

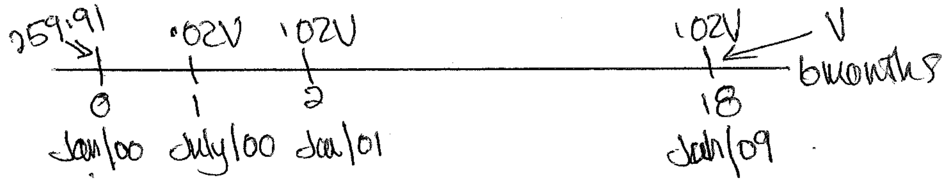
$R_3 \Rightarrow -4z = -100{,}000 \Rightarrow z = 25{,}000$ (1)

$R_2 \Rightarrow -y + 50{,}000 = 0 \Rightarrow y = 50{,}000$ (1)

$R_1 \Rightarrow x + 50{,}000 + 25{,}000 = 100{,}000 \Rightarrow x = 25{,}000$ (1)

should invest \$25,000 in bonds & mortgages & \$50,000 in cerficates of deposit.

15. (a) A bond sold for \$259.91 on January 1, 2000. It had semi-annual interest payments (the next one on July 1, 2000) at a semi-annual coupon rate of 2% and matured on January 1, 2009. The semi-annual yield rate on January 1, 2000 was 2.5%. What was the face value of the bond to the nearest dollar?



(5) $\text{Price} = 259.91 = .02V\,a_{\overline{18}|.025} + V(1.025)^{-18}$ (2)

$= V(.9282333182)$

$\therefore V = \$280$ (3)

(b) On January 1, 2003, the same bond sold for \$287.52. Find the semi-annual yield rate at that time. (you may stop when the price is within \$2 of the actual price)

$\text{Price} = 280(.02)a_{\overline{12}|i} + 280(1+i)^{-12}$ (1)

since price > 280 $\quad i < r = .02$ (1)

students may do any of the following:

(8) $i = 0.015 \Rightarrow \text{Price} = 295.27$

$i = 0.016 \Rightarrow \text{Price} = 292.14$

$i = 0.017 \Rightarrow \text{Price} = 289.05$ (6)

$i = 0.0175 \Rightarrow \text{Price} = 287.52$ ← actual

$i = 0.018 \Rightarrow \text{Price} = 286$

$i = 0.019 \Rightarrow \text{Price} = 282.98$.

Any answer between 1.7% & 1.8% will do.