MAT 133Y1Y TERM TEST 1

Thursday, 5 June, 2014, 6:10 pm – 8:00 pm

Code 1

FAMILY NAME Solutions

GIVEN NAME(S)

STUDENT NO.

SIGNATURE

GRADER'S REPORT	
Question	Mark
MC/40	
B1/15	
B2/15	
B3/15	
B4/15	
TOTAL	

NOTE:

1. **Aids Allowed:** Calculator with empty memory, to be supplied by student. Absolutely no graphing calculators allowed.
2. **Instructions:** Fill in the information on this page and ensure that the test contains 10 pages.
3. This test has 10 multiple choice questions worth 4 marks each and 4 written-answer questions worth 15 marks each.
   For the **multiple choice questions** indicate your answers by circling the appropriate letters (**A**, **B**, **C**, **D**, or **E**) on **this page (page 1)**. A multiple choice question left blank on **this page**, or having an incorrect answer circled, or having more than one answer circled, will be assigned a mark of 0. For the **written-answer questions**, present your solutions in the spaces provided. Use the multiple choice question pages or the back of any of the pages for rough work, for any of the questions.

ANSWER BOX FOR PART A

Circle the correct answer.

1.	A	B	C	D	E
2.	A	B	C	D	E
3.	A	B	C	D	E
4.	A	B	C	D	E
5.	A	B	C	D	E
6.	A	B	C	D	E
7.	A	B	C	D	E
8.	A	B	C	D	E
9.	A	B	C	D	E
10.	A	B	C	D	E

PART A. Multiple Choice

1. *[4 marks]*

If the nominal rate of interest is 8% per year, compounded quarterly, then the effective annual rate of interest is

A 5.41%

B 2.71%

C 8.00%

D 8.24%

E 3.61%

$$\left(1+\frac{8\%}{4}\right)^4-1$$

$$=(1.02)^4-1$$

2. *[4 marks]*

A person borrows \$5,000 today at 6% compounded semiannually. If he agrees to pay \$1,000 one year from now, \$2,000 two years from now, and \$$x$ three years from now, then \$$x$ =

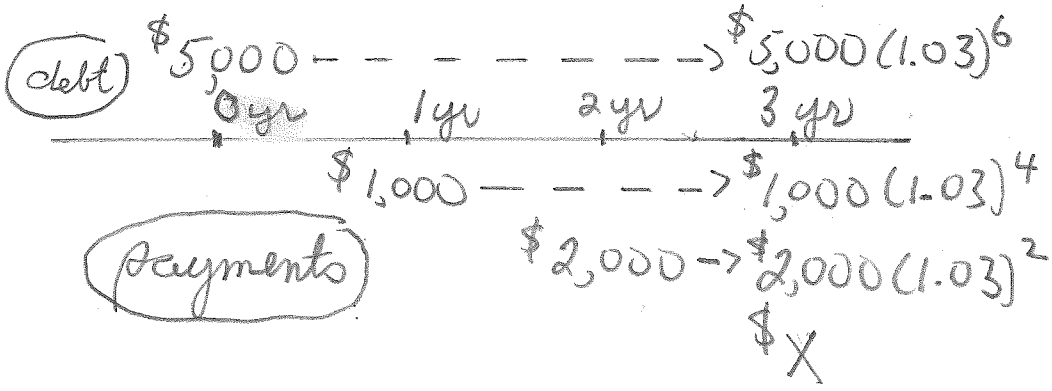
A \$2,722.95

B \$2,000

C \$2,572.60

D \$2,121.80

E \$2,120



$$X+2000(1.03)^2+1000(1.03)^4=5000(1.03)^6$$

$$X=5000(1.03)^6-1000(1.03)^4-2000(1.03)^2$$

3. *[4 marks]*

Mr. Hood buys an ordinary annuity with annual payments of \$10,000, which will pay for 20 years and be donated to charity as long as Mr. Hood lives. Just after the 10^{th} payment and charity donation, Mr. Hood dies, and his heirs cash in the remainder of the annuity. Assuming that the interest rate being used is 5% compounded annually, how much do the heirs get?

A \$124,622.10

B \$125,778.93

C \$77,217.35

D \$83,064.14

E \$100,000

$\$10{,}000\,\dfrac{1-(1.05)^{-10}}{.05}$

4. *[4 marks]*

A sinking fund, with equal quarterly payments at the end of each quarter for 5 years, must accumulate to \$2,000,000 at the end. If the payments are being invested at a nominal rate of 6% compounded monthly, then the quarterly payments (to the nearest dollar) are

A \$91,413

B \$100,000

C \$33,333

D \$116,578

E \$86,428

Let \$X = quarterly payment,
r = effective quarterly interest
$1+r = \left(1+\frac{6\%}{12}\right)^3 = (1.005)^3$

$X\,\dfrac{(1+r)^{20}-1}{r} = 2\cdot 10^6$ so $X = 2\cdot 10^6\,\dfrac{(1.005)^3-1}{(1.005)^{60}-1}$

5. *[4 marks]*

A \$100 bond has 11 years to maturity, semiannual coupons with annual coupon rate 9.5%, and annual yield rate 8.5%. If a coupon has just been redeemed, what is its market price?

A \$106.97

B \$103.25

C \$101.00

D \$107.06

E \$ 99.00

$\$100(1.0425)^{-22} + \$4.75\dfrac{1-(1.0425)^{-22}}{.0425}$

1 coupon: $\$100 \cdot \frac{1}{2} \cdot 9.5\%$

semiannual yield $= \frac{1}{2} \cdot 8.5\%$

6. *[4 marks]*

the easier of 2 possible groupings

$$\begin{bmatrix}1\\4\end{bmatrix}\left(\begin{bmatrix}3\\-1\end{bmatrix}^T\begin{bmatrix}-2 & 7\\2 & 0\end{bmatrix}\right) - 3\begin{bmatrix}2 & -1\\-5 & 4\end{bmatrix}$$

A equals $\begin{bmatrix}-14 & -24\\17 & 72\end{bmatrix}$

B equals $\begin{bmatrix}-14 & 24\\-17 & 72\end{bmatrix}$

C equals $\begin{bmatrix}14 & 24\\-17 & 72\end{bmatrix}$

D is not defined

E equals $\begin{bmatrix}-14 & 24\\-17 & -72\end{bmatrix}$

$\begin{bmatrix}3 & -1\end{bmatrix}\begin{bmatrix}-2 & 7\\2 & 0\end{bmatrix} = \begin{bmatrix}-8 & 21\end{bmatrix}$

$\begin{bmatrix}1\\4\end{bmatrix}\begin{bmatrix}-8 & 21\end{bmatrix} = \begin{bmatrix}-8 & 21\\-32 & 84\end{bmatrix}$

$\begin{bmatrix}-8 & 21\\-32 & 84\end{bmatrix} - \begin{bmatrix}6 & -3\\-15 & 12\end{bmatrix}$

(Note: If M is any matrix, M^T denotes the transpose of M.)

7. *[4 marks]*

Let A, B be two $n \times n$ matrices. Which of the following is **not** always true?

A $(A - A^T)^T = A^T - A$

B $(A + B)^T = B^T + A^T$

C $(AB)^3 = A(BA)^2B$

D $(A + B)^2 = A^2 + 2AB + B^2$

E $(AB)^T = B^T A^T$

$(A+B)^2 = A^2 + AB + BA + B^2$

but $AB \neq BA$ in general

(Note: if M is any matrix, M^T denotes the transpose of M.)

8. *[4 marks]*

Given:

$$\begin{array}{rcrcrcl} x & - & 7y & + & z & = & 3 \\ 2x & - & 14y & + & 3z & = & 4 \end{array}$$

All solutions of this system are given by:

A $z = -2,\ x \in \mathbb{R},\ y = 7x - 5$

B $z \in \mathbb{R},\ y \in \mathbb{R},\ x = 5 + 7y$

C $z = -2,\ y \in \mathbb{R},\ x = 5 + 7y$

D $z \in \mathbb{R},\ x \in \mathbb{R},\ y = (x - 5)/7$

E $z = -2,\ y = 0,\ x = 5$

$$\left[\begin{array}{ccc|c} 1 & -7 & 1 & 3 \\ 2 & -14 & 3 & 4 \end{array}\right]$$

$$\simeq \left[\begin{array}{ccc|c} 1 & -7 & 1 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\simeq \left[\begin{array}{ccc|c} 1 & -7 & 0 & 5 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

9. *[4 marks]*

A homogeneous system of linear equations with 4 equations and 9 unknowns **must** have

A a family of solutions with exactly 5 parameters

B a family of solutions with at least 5 parameters

C a family of solutions with at most 4 parameters

D no solutions

E a family of solutions with exactly 4 parameters

Row reduction will produce an augmented matrix having no more than 4 leading elements (and fewer in case of a row of zeros).

10. *[4 marks]*

If I denotes the 3×3 identity matrix and A is a 3×3 matrix such that $(A+3I)(A-2I) = 3A - 4I$ then A^{-1}

A equals $\frac{1}{2}A - I$

B equals $\frac{1}{2}(A - I)$

C does not exist

D equals $A + \frac{1}{2}I$

E equals $-\frac{1}{2}A + I$

$$A^2 + A - 6I = 3A - 4I$$

$$A^2 - 2A = 2I$$

$$A \cdot \frac{1}{2}A - A = I$$

$$A\left(\frac{1}{2}A - I\right) = I$$

PART B. Written-Answer Questions
SHOW YOUR WORK.

B1. *[15 marks]*

A man makes annual deposits of \$1,000 into a fund paying a nominal rate of 5% compounded annually, starting on his 45^{th} birthday and continuing until his 64^{th} birthday. Beginning on his 65^{th} birthday, he plans to make annual withdrawals of \$2,000.

B1.(a) *[6 marks]*

To the nearest cent, how much is in the fund on his 64^{th} birthday?

20 deposits. $\$1{,}000\,\frac{(1.05)^{20}-1}{.05} = \$33{,}065.95$

B1.(b) *[9 marks]*

How many full withdrawals of \$2,000 each can he make?

Let n = the number of withdrawals.

$1000\,s_{\overline{20}|.05} = 2000\,a_{\overline{n}|.05}$ so that

$\frac{1}{2}((1.05)^{20}-1) = 1-(1.05)^{-n}$, $(1.05)^{-n} = \frac{3-(1.05)^{20}}{2}$,

$-n\ln(1.05) = \ln\left(\frac{3-(1.05)^{20}}{2}\right)$, and

$n = \dfrac{-\ln\left(\frac{3-(1.05)^{20}}{2}\right)}{\ln(1.05)} = 35.9\ldots$

35 full withdrawals

B2. *[15 marks]*

A \$70,000, 12 year mortgage, at 8% nominal annual interest compounded semiannually is taken out on 1 January, 1991. The first payment is on 1 February, 1991, and thereafter payments are monthly.

B2.(a) *[7 marks]*

Find the amount of the monthly payment, to the nearest cent.

Let \$X = the monthly payment, r = effective monthly interest; $1+r = (1.04)^{\frac{1}{6}}$.

$\$X\, a_{\overline{144}|r} = \$70000$, so

$$\$X = \$70000\,\frac{(1.04)^{\frac{1}{6}}-1}{1-(1.04)^{-24}} = \boxed{\$752.73}$$

B2.(b) *[8 marks]*

To the nearest cent, find the interest included in the payment of 1 February, 2002.

12 year mortgage $\Rightarrow$ last payment is on 1 Jan., 2003. Just after 1 Jan., 2002, 12 payments remain. Principal outstanding is $\$X\, a_{\overline{12}|r}$ and interest paid on 1 Feb., 2000 is

$$\$X\, a_{\overline{12}|r} \cdot r = \$X\left(1-(1.04)^{-2}\right) = \boxed{\$56.79}$$

B3.(a) *[9 marks]*

Find the inverse of the matrix $\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -2 \\ 1 & -3 & 0 \end{bmatrix}$. Show your work.

$$\left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 1 & 0 & 0 \\ 1 & 0 & -2 & 0 & 1 & 0 \\ 1 & -3 & 0 & 0 & 0 & 1 \end{array}\right] \simeq \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 1 & -3 & 0 & 0 & 0 & 1 \end{array}\right] \leftarrow 5$$

$$\simeq \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & -3 & 2 & 0 & -1 & 1 \end{array}\right] \simeq \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -3 & -1 & 1 \end{array}\right]$$

$$\simeq \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 6 & 3 & -2 \\ 0 & 1 & 0 & 2 & 1 & -1 \\ 0 & 0 & 1 & 3 & 1 & -1 \end{array}\right] \quad \text{Inverse} = \begin{bmatrix} 6 & 3 & -2 \\ 2 & 1 & -1 \\ 3 & 1 & -1 \end{bmatrix}$$

B3.(b) *[6 marks]*

Use your solution to part (a) to solve the system below. **No marks** will be assigned to another method of solution. Show your work.

$$\begin{array}{rcrcrcl} & - & y & + & z & = & 1 \\ x & & & - & 2z & = & 3 \\ x & - & 3y & & & = & 4 \end{array}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 & 3 & -2 \\ 2 & 1 & -1 \\ 3 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 2 \end{bmatrix}$$

B4. *[15 marks]*

A box of candies contains cinnamon hearts, licorice sticks, and chocolates — 16 candies altogether. Each cinnamon heart costs 1 cent and weighs 2 grams, each licorice stick costs 3 cents and weighs 5 grams, and each chocolate costs 7 cents and weighs 9 grams. If the total cost of the candies in the box is 46 cents and their total weight is 69 grams, find how many of each type of candy the box contains.

Let x = number of cinnamon hearts,
y = number of licorice sticks,
z = number of chocolates.

Then:

number of candies $= x + y + z = 16$

total cost $= x + 3y + 7z = 46$

total weight $= 2x + 5y + 9z = 69$

Solution by row reduction:

$$\begin{bmatrix} \textcircled{1} & 1 & 1 & 16 \\ 1 & 3 & 7 & 46 \\ 2 & 5 & 9 & 69 \end{bmatrix} \simeq \begin{bmatrix} 1 & 1 & 1 & 16 \\ 0 & \textcircled{2} & 6 & 30 \\ 0 & 3 & 7 & 37 \end{bmatrix}$$

$$\simeq \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 3 & 15 \\ 0 & 0 & \textcircled{-2} & -8 \end{bmatrix} \simeq \begin{bmatrix} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

9 cinnamon hearts, 3 licorice sticks, 4 chocolates