MAT 133Y1Y TERM TEST #1

THURSDAY, JUNE 4, 2009 7:30 - 9:30 PM

FAMILY NAME: ______________________________

GIVEN NAMES: ______________________________

STUDENT NUMBER: ______________________________

TUTORIAL ROOM: ______________________________

Aids Allowed: Calculator with empty memory, to be supplied by the student. Absolutely no graphing calculators allowed.

Instructions: This test has 10 multiple choice questions worth 4 marks each and 5 written answer questions worth a total of 60 marks. For each multiple choice question, you may do your rough work in the test booklet, but you must record your answer by circling one of the letters A, B, C, D or E which appear on the front page of the test. A multiple choice question left blank, or having an incorrect answer circled, or having more than one answer circled, will be assigned a mark of 0. For the written answer solutions, present your solutions in the spaces provided. Use the back of the question pages for your rough work.

GRADER'S REPORT	
Multiple Choice	/ 40
Question 11	/ 15
Question 12	/ 15
Question 13	/ 15
Question 14	/ 15
TOTAL	/100

ANSWERS FOR MULTIPLE CHOICE
Circle the correct answer

1.	**A**	**B**	**C**	**D**	**E**
2.	**A**	**B**	**C**	**D**	**E**
3.	**A**	**B**	**C**	**D**	**E**
4.	**A**	**B**	**C**	**D**	**E**
5.	**A**	**B**	**C**	**D**	**E**
6.	**A**	**B**	**C**	**D**	**E**
7.	**A**	**B**	**C**	**D**	**E**
8.	**A**	**B**	**C**	**D**	**E**
9.	**A**	**B**	**C**	**D**	**E**
10.	**A**	**B**	**C**	**D**	**E**

(r)

1. The annual rate compounded continuously that is equivalent to a nominal rate of 3 % compounded every 4 months is closest to:

A 2.989%

B 2.9%

C 3.03%

D 3.045 %

E 2.985%

$$e^r = (1.01)^3$$

$$r = 3\ln(1.01)$$

$$\approx 0.029850993$$

2. If Peter borrows \$200 at the end of 2 years and \$500 at the end of 5 years, then how much will he owe at the end of 10 years, if he is charged interest at an effective annual rate of 4%?

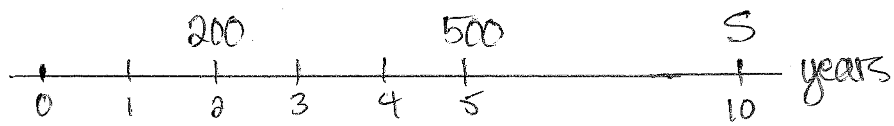
A \$595.87

B \$882.04

C \$1036.17

D \$4551.01

E \$928.00

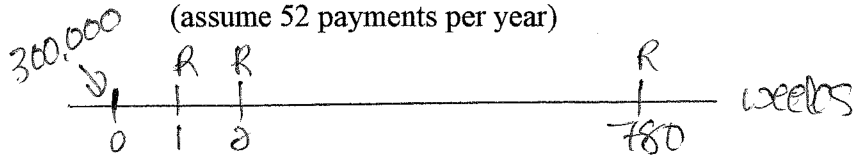


$$S = 200(1.04)^8 + 500(1.04)^5$$

$$\approx 273.71 + 608.33$$

$$= 882.04$$

3. A \$300,000 15 year Mortgage has weekly payments, the first payment due in one week. If the interest rate charged on the mortgage is 5% compounded weekly, then the weekly payment is closest to: (assume 52 payments per year)



A \$548.56

B \$931.92

C \$384.62

D \$546.88

E \$384.98

weekly rate $= r = \frac{.05}{52} \approx .000961538$

$R\,a_{\overline{780}|r} = 300{,}000$.

$R \approx 546.8844$.

4. A loan is amortized over 10 years and has payments of \$200 at the end of each month. If interest is charged at the rate of 6% compounded monthly, then the amount of interest paid in the 100^{th} payment is:

A \$78.54

B \$19.89

C \$10.00

D \$12.00

E \$18.98

Outstanding Principal after 99th payment

= present value of 21 remaining payments

$= 200\,a_{\overline{21}|.005}$

≈ 3977.59585

Interest in 100th payment is

$3977.59585 \times .005 \approx 19.887979$

5. A bond of face value $500 sells for $467.36 and has semi-annual interest payments of $10 (the next one in 6 months time). If the semi-annual yield rate is 2.5%, then how many interest payments are there left?

A 20

B 15

C 16

D 14

E 10

$$10\,a_{\overline{n}|.025} + 500(1.025)^{-n} = 467.36$$

$$10\left[\frac{1-(1.025)^{-n}}{.025}\right] + 500(1.025)^{-n} = 467.36$$

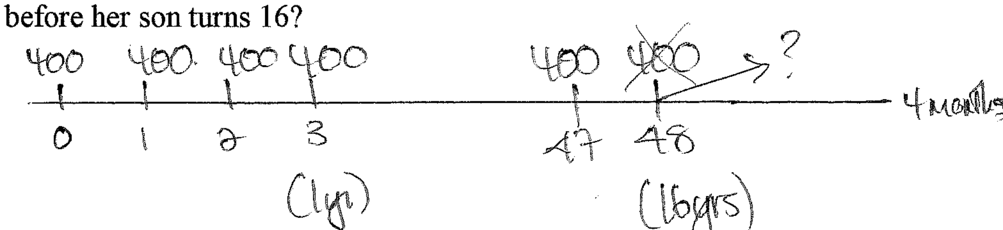
$$400 - 400(1.025)^{-n} + 500(1.025)^{-n} = 467.36$$

$$100(1.025)^{-n} = 67.36.$$

$(1.025)^{-n} = .6736$ $\qquad -n\ln(1.025) = \ln(.6736)$

$$n = \frac{\ln(.6736)}{-\ln(1.025)} \approx 16$$

6. When her son is born, a mother deposits $400 into a savings account paying 6% compounded 3 times per year and continues to deposit $400 every 4 months thererafter. How much money will there be in the account just before her son turns 16?



A $32,376.24

B $32,776.24

C $31,741.41

D $19,200.00

E $49,671.75

$$\text{Amount} = 400\,s_{\overline{49}|.02} - 400$$

$$\approx 32{,}376.23586.$$

7. In the matrix equation: $\begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} X = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ where X is a 2x3 matrix, then

X =

A $\begin{bmatrix} 2 & -3 & 2 \\ 1 & -1 & 1 \end{bmatrix}$

B $\begin{bmatrix} -2 & 6 & -2 \\ -1 & 2 & -1 \end{bmatrix}$

C $\begin{bmatrix} -1 & 3 & -1 \\ -1 & 2 & -1 \end{bmatrix}$

D $\begin{bmatrix} 1/2 & 0 & -1/3 \\ 1 & -1 & 0 \end{bmatrix}$

E $\begin{bmatrix} -1 & 3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$

$$X = \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 & -1 \\ -1 & 2 & -1 \end{bmatrix}$$

ie $\left[\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 0 & -1 & 1 & -2 \end{array}\right] R_1 - 2R_2$

$\Rightarrow \left[\begin{array}{cc|cc} -2 & 0 & 2 & -6 \\ 0 & -1 & 1 & -2 \end{array}\right] 3R_2 - R_1$

$\Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & -1 & 3 \\ 0 & 1 & -1 & 2 \end{array}\right] \begin{array}{l} R_1 \div -2 \\ -R_2 \end{array}$

8. If A is a 2x3 matrix and B is a 3x2 matrix and C is a 2x2 matrix, then which of the following operations is not defined?

A A x (B x C)

B (A x B) + C^T

C (B x C^2) + A^T

D A^T x (C x B^T)

E $(C \times A)^T$ x B $= [(2\times2)\times(2\times3)]^T \times (3\times2)$

$= [2\times3]^T \times (3\times2)$

$= (3\times2)\times(3\times2)$

impossible!

9. All solutions of the linear system:

$$\begin{aligned} x - 7y + z &= 3 \\ 2x - 14y + 3z &= 4 \end{aligned}$$

are given by:

$$\left[\begin{array}{ccc|c} 1 & -7 & 1 & 3 \\ 2 & -14 & 3 & 4 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -7 & 1 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right] R_2 - 2R_1$$

$R_2 \Rightarrow z = -2$.

$R_1 \Rightarrow x - 7y - 2 = 3$

$\Rightarrow x = 5 + 7y \quad y \in R$.

A $z \in R,\ y \in R,\ x = 5 + 7y$

B $z \in R,\ x \in R,\ y = (x - 5)/7$

C $z = -2,\ x \in R,\ y = 7x - 5$

D $z = -2,\ y \in R,\ x = 5 + 7y$

E no solution

10. The system given by:

$$\begin{aligned} x + y - 3z &= 5 \\ x - 3y + z &= -7 \\ 2x - y - 3z &= 2 \end{aligned}$$

has:

$$\left[\begin{array}{ccc|c} 1 & 1 & -3 & 5 \\ 1 & -3 & 1 & -7 \\ 2 & -1 & -3 & 2 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -3 & 5 \\ 0 & -4 & 4 & -12 \\ 0 & -3 & 3 & -8 \end{array}\right] \begin{array}{l} \\ R_2 - R_1 \\ R_3 - 2R_1 \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -3 & 5 \\ 0 & -1 & 1 & -3 \\ 0 & -3 & 3 & -8 \end{array}\right] R_2 \div 4$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -3 & 5 \\ 0 & -1 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{array}\right] R_3 - 3R_2$$

A no solution

B a unique solution

C infinitely many solutions with $x = 0$

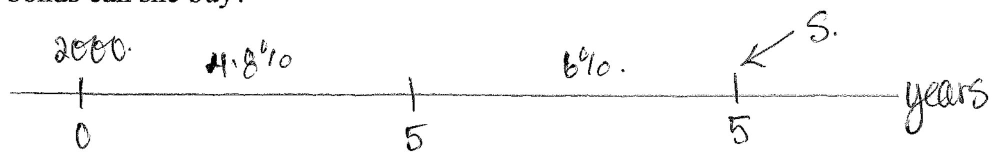
D infinitely many solutions with $y = 0$

E infinitely many solutions with $z = 0$

$R_3 \Rightarrow 0x + 0y + 0z = 1 \quad \therefore$ no solution.

11. Emily deposits \$2000 in an account which earns interest at 4.8% compounded quarterly for the first 5 years and then earns interest at 6% compounded semi-annually for the next 5 years. At the end of 10 years, she takes the money in the account and buys bonds of face value \$100 that have 12 semi-annual interest payments remaining at a semi-annual coupon rate of 3.5%.

(a) If the semi-annual yield rate is 3% at the time that she buys the bonds, then how many full bonds can she buy?



$$S = 2000(1.012)^{20}(1.03)^{10} = \$3412.03. \quad 4$$

$$\text{Price of Bond} = 100(.035)\,a_{\overline{12}|.03} + 100(1.03)^{-12} \quad 3$$
$$\approx \$104.98. \quad 4$$

(13)

$$\#\text{ of bonds} = 3412.03 \div 104.98 = 32.5 \quad 2$$

$\therefore$ she can purchase 32 full bonds.

(b) How much money will she have leftover?

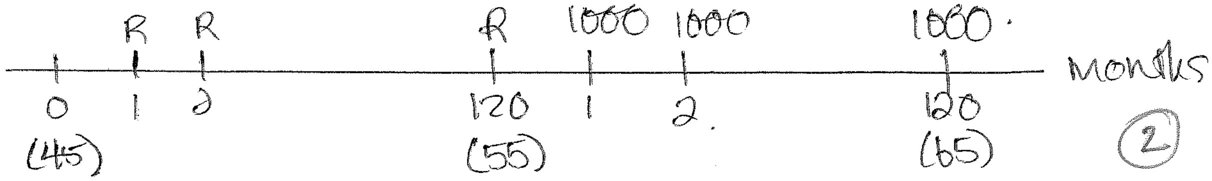
$$32 \times 104.98 = 3359.36. \quad 1$$

(2)

$$3412.03 - 3359.36 = 52.67 \quad 1$$

$\therefore$ she has \$52.67 left.

12. One month after Joseph turns 45, he begins depositing \$R per month into a savings account that earns an effective annual interest rate of 5%. The last deposit he makes is on his 55^{th} birthday, at which time he retires from his job. One month after his 55^{th} birthday, he begins to withdraw \$1000 per month from the account and continues to do so up to and including his 65^{th} birthday. Find the value of R so that there is just enough money in the account to cover the withdrawals.



2

let r = monthly rate

$$(1+r)^{12} = 1.05 \Rightarrow r = (1.05)^{1/12} - 1$$

$$\approx .004074124$$ 4

(15)

$$R\,s_{\overline{120}|r} = 1000\,a_{\overline{120}|r}$$ 4

$$R(154.36) = 94765.59$$

$$R \approx 613.93$$ 5

$\therefore$ The deposits should be \$613.93.

13. Roger invests \$50,000 of his savings in three different investments – Bonds, GIC's and RRSP's. His annual income depends on the rates annual of return he earns from these investments and is given in the table below:

Bonds	GIC's	RRSP's	Annual Income
4%	6%	5%	\$2550
6%	5%	4%	\$2350

If he invests \$x in bonds, \$y in GIC's and \$z in RRSP's, then represent the above information by a linear system of equations and solve the system to find x, y and z.

$$x+y+z = 50000$$
$$.04x+.06y+.05z = 2550$$
$$.06x+.05y+.04z = 2350$$

3

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 50000 \\ 4 & 6 & 5 & 255000 \\ 6 & 5 & 4 & 235000 \end{array}\right]$$

3

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 50000 \\ 0 & 2 & 1 & 55000 \\ 0 & -1 & -2 & -65000 \end{array}\right] \begin{array}{l} \\ R_2-4R_1 \\ R_3-6R_1 \end{array}$$

4

(15)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 50000 \\ 0 & 2 & 1 & 55000 \\ 0 & 0 & -3 & -75000 \end{array}\right] \begin{array}{l} \\ \\ 2R_3+R_2 \end{array}$$

2

$R_3 \Rightarrow -3z = -75000 \Rightarrow z = 25000$ 1

$R_2 \Rightarrow 2y + 25000 = 55000 \Rightarrow y = 15000$ 1

$R_1 \Rightarrow x + 15000 + 25000 = 50000.$

$\Rightarrow x = 10000.$ 1

So He invests \$10,000 in bonds, \$15000 in GIC's and \$25000 in RRSP'S.

14. Given the input-output matrix for the three industries below:

	Industry A	Industry B	Industry C		Final Demand
Industry A	40	0	80	:	80
Industry B	0	10	40	:	50
Industry C	80	20	40	:	60
Other Production Factors	80	70	40	:	–
	200	100	200.	(1)	

Find the new outputs for each of the industries, if the final demand for all three of the Industries changes to 60.

$$A = \begin{bmatrix} \frac{1}{5} & 0 & \frac{2}{5} \\ 0 & \frac{1}{10} & \frac{1}{5} \\ \frac{2}{5} & \frac{1}{5} & \frac{1}{5} \end{bmatrix} \quad (2) \qquad I - A = \begin{bmatrix} \frac{4}{5} & 0 & -\frac{2}{5} \\ 0 & \frac{9}{10} & -\frac{1}{5} \\ -\frac{2}{5} & -\frac{1}{5} & \frac{4}{5} \end{bmatrix} \quad (1) \qquad D = \begin{bmatrix} 60 \\ 60 \\ 60 \end{bmatrix}$$

$$X = \begin{bmatrix} X_A \\ X_B \\ X_C \end{bmatrix}$$

$$(I-A)X = D$$

(15)

$$\left[\begin{array}{ccc|c} \frac{4}{5} & 0 & -\frac{2}{5} & 60 \\ 0 & \frac{9}{10} & -\frac{1}{5} & 60 \\ -\frac{2}{5} & -\frac{1}{5} & \frac{4}{5} & 60 \end{array}\right] (2) \Rightarrow \left[\begin{array}{ccc|c} 4 & 0 & -2 & 300 \\ 0 & 9 & -2 & 600 \\ -2 & -1 & 4 & 300 \end{array}\right] \begin{matrix} 5R_1 \\ 10R_2 \\ 5R_3 \end{matrix} \quad (2)$$

$$\Rightarrow \left[\begin{array}{ccc|c} 4 & 0 & -2 & 300 \\ 0 & 9 & -2 & 600 \\ 0 & -2 & 6 & 900 \end{array}\right] 2R_3 + R_1 \quad (2) \Rightarrow \left[\begin{array}{ccc|c} 2 & 0 & -1 & 150 \\ 0 & 9 & -2 & 600 \\ 0 & 0 & 50 & 9300 \end{array}\right] \begin{matrix} R_1 \div 2 \\ \\ 2R_2 + 9R_3 \end{matrix} \quad (2)$$

$R_3 \Rightarrow 50X_C = 9300 \Rightarrow X_C = 186$ (1)

$R_2 \Rightarrow 9X_B - 2(186) = 600 \Rightarrow X_B = 108$ (1)

$R_1 \Rightarrow 2X_A - 186 = 150 \Rightarrow X_A = 168$ (1)

∴ new outputs for Ind A, B & C are 168, 108 & 186 units respectively.