MAT 133Y1Y TERM TEST #3

THURSDAY, JULY 29, 2010 7:00 - 9:00 PM

FAMILY NAME: ______________________________

GIVEN NAMES: ______________________________

STUDENT NUMBER: ______________________________

TUTORIAL ROOM: ______________________________

Aids Allowed: Calculator with empty memory, to be supplied by the student. Absolutely no graphing calculators allowed.

Instructions: This test has 10 multiple choice questions worth 4 marks each and 5 written answer questions worth a total of 60 marks. For each multiple choice question, you may do your rough work in the test booklet, but you must record your answer by circling one of the letters A, B, C, D or E which appear on the front page of the test. A multiple choice question left blank, or having an incorrect answer circled, or having more than one answer circled, will be assigned a mark of 0. For the written answer solutions, present your solutions in the spaces provided. Use the back of the question pages for your rough work.

GRADER'S REPORT	
Multiple Choice	/ 40
Question 11	/ 15
Question 12	/ 15
Question 13	/ 15
Question 14	/ 15
TOTAL	/100

ANSWERS FOR MULTIPLE CHOICE Circle the correct answer					
1.	**A**	**B**	**C**	**D**	**E**
2.	**A**	**B**	**C**	**D**	**E**
3.	**A**	**B**	**C**	**D**	**E**
4.	**A**	**B**	**C**	**D**	**E**
5.	**A**	**B**	**C**	**D**	**E**
6.	**A**	**B**	**C**	**D**	**E**
7.	**A**	**B**	**C**	**D**	**E**
8.	**A**	**B**	**C**	**D**	**E**
9.	**A**	**B**	**C**	**D**	**E**
10.	**A**	**B**	**C**	**D**	**E**

1. $\int_1^e \frac{4t^3 - 3t^2 + 4t - 1}{t^2}\, dt$ $=$ $\int_1^e \left[4t - 3 + \frac{4}{t} - \frac{1}{t^2}\right] dt$

$= \left[2t^2 - 3t + 4\ln|t| + \frac{1}{t}\right]_1^e$

$= (2e^2 - 3e + 4\ln e + \frac{1}{e})$

$- (2 - 3 + 4\ln 1 + 1)$

$= 2e^2 - 3e + \frac{1}{e} + 4$

A $2e^2 - 3e + 1/e + 4$

B $4e + 4/e - 1/e^2 - 7$

C $3e - 1/e - 2e^2 - 4$

D $3e - 6/e - 3/e^2 - 6$

E $2e^2 - 3e + 1/e$

2. $\lim_{n \to +\infty} \frac{4}{n} \left\{ \frac{2}{1+(4/n)} + \frac{2}{1+2(4/n)} + \frac{2}{1+3(4/n)} + \cdots\cdots + \frac{2}{1+n(4/n)} \right\} =$

$\int_1^5 \frac{2}{x}\, dx$

$= \left[2\ln|x|\right]_1^5$

$= 2\ln 5 - 2\ln 1$

$= 2\ln 5$

A 8

B 2ln5

C ln5

D 2ln4

E 0

3. $\int_0^1 \frac{x-1}{x+1}\,dx$ $= \int_0^1 \left[1 - \frac{2}{x+1}\right] dx$

$$x+1 \overline{)\, x-1 \,}^{\;1} \quad \frac{x+1}{-2}$$

A $1 + 2\ln 2$

B $-1/3$

C $1 - 2\ln 2$

D $-2/7$

E $\frac{-\ln 2}{2}$

$$= \left[x - 2\ln|x+1|\right]_0^1$$

$$= 1 - 2\ln 2 + 2\ln 0$$

$$= 1 - 2\ln 2.$$

4. $\frac{d}{dx}\int_x^2 (e)^{t^2}\,dt$ $= \frac{d}{dx}\left[F(t)\right]_x^2$ where $F'(t) = e^{t^2}$

A $(e)^{x^2} - e^4$

B $(e)^{x^2}$

C $(e)^{x^2}(2x)$

D $-(e)^{x^2}$

E $e^4 - (e)^{x^2}$

$$= \frac{d}{dx}\left[F(2) - F(x)\right]$$

$$= 0 - F'(x)$$

$$= -e^{x^2}$$

5. $\int_0^1 2^{(1-2x)}\,dx =$ $\left[\dfrac{2^{1-2x}}{-2\ln 2}\right]_0^1 = \dfrac{2^{-1}}{-2\ln 2} - \dfrac{2^{1}}{-2\ln 2}$

$= \dfrac{-\frac{3}{2}}{-2\ln 2}$

$= \dfrac{3}{4\ln 2}$.

A -3/2

B 3/4

C - 3/(2ln2)

D -5/4

E 3/(4ln2)

6. The average value of $f(x) = x\sqrt{x^2 + 16}$ on the interval [0 , 3] is:

A 61/9

B 125/9

C 61/6

D 125/6

E 61/3

$\bar{f} = \frac{1}{3}\int_0^3 x\sqrt{x^2+16}\,dx$

$= \frac{1}{6}\int_{16}^{25} (u)^{1/2}\,du$.

$u = x^2+16$

$du = 2x\,dx$

$x=0 \Rightarrow u=16$

$x=3 \Rightarrow u=25$

$= \left[\frac{1}{6}\,\frac{u^{3/2}}{\frac{3}{2}}\right]_{16}^{25}$

$= \frac{1}{9}\left[125-64\right] = \frac{61}{9}$

7. The Marginal Cost for a product is given by: $\frac{dc}{dq} = q^2 + 2q + 7$

If fixed cost is $10,000 , then when 100 units are produced the total cost is:

A $350,033

$C = \frac{q^3}{3} + q^2 + 7q + C_0$

B $354,033

$q = 0 \Rightarrow C = 10,000 \Rightarrow C_0 = 10,000$

C $345,334

$\therefore C = \frac{1}{3}q^3 + q^2 + 7q + 10,000$

D $345,333

$q = 100 \Rightarrow C = \frac{1000000}{3} + 10000$

E $382,382

$+ 700 + 10000$

$\hat{=} 354033$

8. If the demand equation for a product is: $p = 800 - 2q^2$ and the supply equation is: $p = 10q + 200$, then the Consumers' Surplus at market equilibrium is given by:

Equilibrium:

A $\int_0^{15} (450 - 2q^2)\, dq$

$10q + 200 = 800 - 2q^2$

B $\int_0^{350} (785 - 2q^2)\, dq$

$2q^2 + 10q - 600 = 0$

$q^2 + 5q - 300 = 0$

C $\int_0^{15} (150 - 10q)\, dq$

$(q + 20)(q - 15) = 0$

$\cancel{q = -20}$ or $q = 15$

D $\int_0^{20} (400 - 2q^2)\, dq$

$q_0 = 15 \Rightarrow p_0 = 350$

E $\int_0^{15} (2q^2 - 450)\, dq$

$CS = \int_0^{15} [800 - 2q^2 - 350]\, dq$

9. If 80 people are put on an island and in 5 years there are 120 people, then how many people will there be in 12 years assuming exponential growth?

let P = population at time t

$P = P_0 e^{kt}$

$P_0 = 80 \quad t = 5 \Rightarrow P = 120$

$\Rightarrow 120 = 80 e^{5k}$

$e^{5k} = \frac{3}{2}$

$P = 80 \left(\frac{3}{2}\right)^{t/5}$

$t = 12 \Rightarrow P = 80\left(\frac{3}{2}\right)^{12/5}$

≈ 211.7

A 176

B 270

C 211

D 200

E 216

10. $\int_0^{+\infty} e^{(-3x)}\, dx =$

$= \lim_{b \to +\infty} \int_0^b e^{-3x}\, dx$

$= \lim_{b \to +\infty} \left[\frac{e^{-3x}}{-3} \right]_0^b$

$= \lim_{b \to +\infty} \frac{e^{-3b}}{-3} + \frac{e^0}{3}$

$\searrow 0$

$= \frac{1}{3}$

A − 1/3

B 1

C 1/3

D − 1

E $+\infty$

11. Find the present value of a continuous annuity with interest at an annual rate of 5% compounded continuously for 25 years, if the payment (in dollars) at time t is at the annual rate of 20t.

$$P.V = \int_0^{25} 20te^{-.05t}\,dt \quad 4$$

$$f(t) = 20t \Rightarrow f'(t) = 20 \quad 4$$

$$g'(t) = e^{-.05t} \Rightarrow g(t) = \frac{e^{-0.05t}}{-0.05}$$

$$= \left[\frac{20te^{-0.05t}}{-0.05} - \int \frac{20e^{-0.05t}}{-0.05}\,dt \right]_0^{25} \quad 2$$

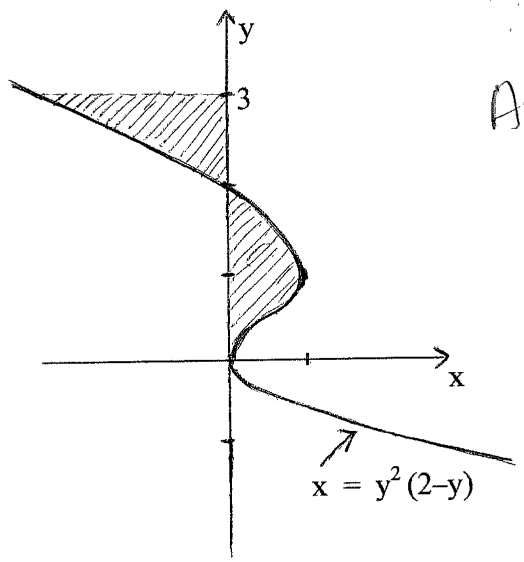
$$= \left[-400te^{-0.05t} - 8000e^{-0.05t} \right]_0^{25} \quad 2$$

(15) $$= \left[-10{,}000e^{-1.25} - 8000e^{-1.25} + 8000 \right]$$

$$= \$2842.91 \quad 3$$

12. Write the area of the shaded regions below in terms of definite integral(s).
Do not evaluate the integrals.

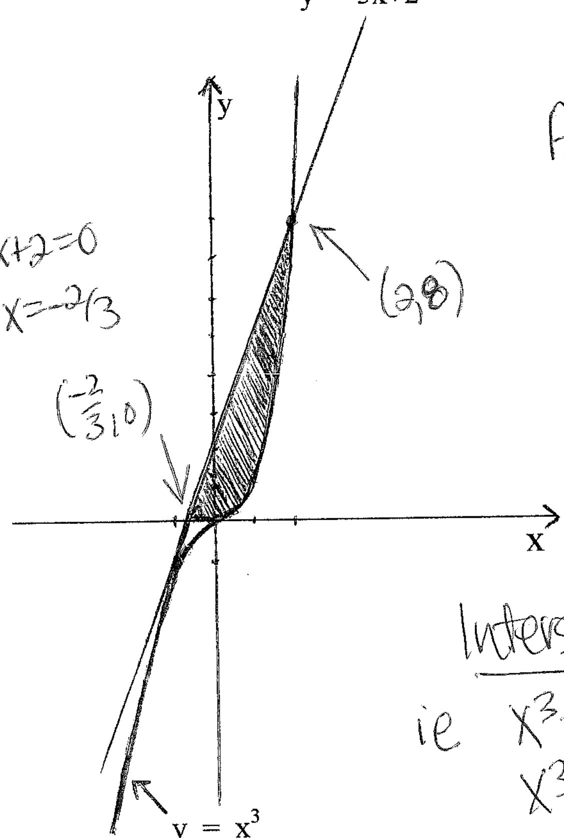
(a)



(6)

$$\text{Area} = \int_0^2 y^2(2-y)\,dy \quad 3$$
$$+ \int_2^3 [0 - y^2(2-y)]\,dy \quad 3$$
$$= \int_0^2 y^2(2-y)\,dy + \int_2^3 y^2(y-2)\,dy.$$

(b)



ie $3x+2=0$
$\Rightarrow x = -2/3$

(9)

$$\text{Area} = \int_{-2/3}^{0} (3x+2)\,dx \quad 3$$
$$+ \int_0^2 (3x+2-x^3)\,dx \quad 3$$

3 — for finding bounds.

Intersection.
ie $x^3 = 3x+2$
$x^3 - 3x - 2 = 0$
$(x-2)(x+1)^2 = 0$

enough to verify $x=2, y=8$. satisfies both curves.

can do by long division.

13. Find the area of the region between the curve $f(x) = \dfrac{1}{x^2(x-1)}$ and the x-axis, from $x = 2$ to $x = 3$. (Leave answer exact – no decimals)

$$\text{Area} = \int_2^3 \frac{1}{x^2(x-1)}\,dx \quad 2 \quad = \int_2^3 \left[\frac{-1}{x} - \frac{1}{x^2} + \frac{1}{x-1}\right] dx \quad 2$$

$$= \left[-\ln|x| + \frac{1}{x} + \ln|x-1|\right]_2^3 \quad 3$$

$$= \left(-\ln 3 + \frac{1}{3} + \ln 2 + \ln 2 - \frac{1}{2} - \ln 1\right)$$

(15) $$= 2\ln 2 - \ln 3 - \frac{1}{6} \quad 3$$

ie $$\frac{1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-1)} = \frac{Ax(x-1) + B(x-1) + Cx^2}{x^2(x-1)}$$

$x = 0 \Rightarrow -B = 1 \Rightarrow B = -1$

$x = 1 \Rightarrow C = 1$ 5

$x = -1 \Rightarrow 2A - 2(-1) + (1) = 1$

$\Rightarrow A = -1$

14. Given: $\frac{dy}{dx} = \frac{x^2 (e^{x^3})}{y^2}$ and when $x = 0$, $y = 2$. Write y <u>explicitly</u> in terms of x.

2

$$\int y^2 dy = \int x^2 e^{x^3} dx = \frac{1}{3}\int e^{x^3} \cdot 3x^2 dx$$

$u = x^3$

$du = 3x^2 dx$

$$\frac{y^3}{3} = \frac{1}{3}\int e^u du$$

1

$$= \frac{1}{3} e^u + C$$

$$= \frac{1}{3} e^{x^3} + C \quad 6$$

(15) $x=0, y=2 \Rightarrow \frac{8}{3} = \frac{e^0}{3} + C \Rightarrow C = \frac{7}{3}$ 3

$$\therefore \frac{y^3}{3} = \frac{e^{x^3}}{3} + \frac{7}{3}$$

$$\Rightarrow y^3 = e^{x^3} + 7$$

$$\Rightarrow y = \sqrt[3]{e^{x^3} + 7}. \quad 3.$$