MAT 133Y1Y TERM TEST #3

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THURSDAY, JULY 28, 2011 7:10 - 9:10 PM

FAMILY NAME: ______________________________

GIVEN NAMES: ______________________________

STUDENT NUMBER: ______________________________

TUTORIAL ROOM: ______________________________

Aids Allowed: Calculator with empty memory, to be supplied by the student. Absolutely no graphing calculators allowed.

Instructions: This test has 10 multiple choice questions worth 4 marks each and 4 written answer questions worth 15 marks each. For each multiple choice question, you may do your rough work in the test booklet, but you must record your answer by circling one of the letters A, B, C, D or E which appear on the front page of the test. A multiple choice question left blank, or having an incorrect answer circled, or having more than one answer circled, will be assigned a mark of 0. For the written answer solutions, present your solutions in the spaces provided. Use the back of the question pages for your rough work.

GRADER'S REPORT	
Multiple Choice	/ 40
Question 11	/ 15
Question 12	/ 15
Question 13	/ 15
Question 14	/ 15
TOTAL	/100

ANSWERS FOR MULTIPLE CHOICE Circle the correct answer					
1.	A	B	C	D	E
2.	A	B	C	D	E
3.	A	B	C	D	E
4.	A	B	C	D	E
5.	A	B	C	D	E
6.	A	B	C	D	E
7.	A	B	C	D	E
8.	A	B	C	D	E
9.	A	B	C	D	E
10.	A	B	C	D	E

1. $$\int_1^4 \frac{3x+1}{\sqrt{x}}\,dx = \int_1^4 [3x^{1/2}+x^{-1/2}]\,dx$$

$$= \left[\frac{3x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2}\right]_1^4$$

$$= \left[2x^{3/2}+2x^{1/2}\right]_1^4$$

$$= [2\times 8 + 4 - 2 - 2]$$

$$= 16$$

A $4\sqrt{2}$

B $-5/16$

C $13/2$

D $5/2$

E 16

2. $$\lim_{n \to +\infty} \frac{5}{n}\left\{ e^{(5/n)} + e^{2(5/n)} + e^{3(5/n)} + \cdots\cdots + e^{n(5/n)} \right\} = \int_0^5 e^x\,dx$$

$$= \left[e^x\right]_0^5$$

$$= e^5 - e^0$$

$$= e^5 - 1$$

A 0

B $e^5 - 1$

C 5

D e^5

E $+\infty$

3. $\displaystyle\int \frac{x^3+3x^2}{x+1}\,dx \quad =$

$$\begin{array}{r} x^2+2x-2 \\ x+1 \overline{)\, x^3+3x^2} \\ \underline{x^3+x^2} \\ 2x^2 \\ \underline{2x^2+2x} \\ -2x \\ \underline{-2x-2} \\ 2 \end{array}$$

A $x^3/3 + x^2 - 2x + 2\ln|x+1| + C$

B $x^4/2 + 3x^3 + C$

C $4x^3/3 + C$

D $x^3/3 + x^2 - 2x + 2 + C$

E $x^4/3 + (2x^3/3)\ln|x+1| + C$

$$= \int \left(x^2+2x-2+\frac{2}{x+1}\right)dx$$

$$= \frac{x^3}{3}+x^2-2x+2\ln|x+1|+C$$

4. $\displaystyle\int_0^{+\infty} \frac{1}{(t-1)^2}\,dt \quad = \lim_{b\to+\infty}\int_0^b \frac{1}{(t-1)^2}\,dt = \lim_{b\to+\infty}\left[\frac{-1}{t-1}\right]_0^b$

$$= \lim_{b\to+\infty}\left[\underbrace{-\frac{1}{b-1}}_{0}+\frac{1}{-1}\right]$$

$$= -1$$

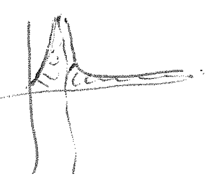
A -2

B 0

C $-1/3$

D -1

E $+\infty$



Asymptote

$+\infty$

5. $\int_1^e \ln x \, dx =$ $\left[x\ln x - \int \frac{1}{x}\cdot x\,dx\right]_1^e$ $\quad f(x)=\ln x \Rightarrow f'(x)=\frac{1}{x}$
$g'(x)=1 \Rightarrow g(x)=x$.

A $\frac{1}{e} - 1$ $\quad = \left[x\ln x - x\right]_1^e$

B $\frac{1}{2}$ $\quad = e\ln e - e - \ln 1 + 1$

C $\frac{e}{2}$ $\quad = 1$

D 1

E $\frac{e-1}{2}$

6. The average value of $f(x) = x^2 \cdot 2^{(x^3)}$ on the interval $[1, 3]$ is:

A $\frac{2^{27} - 2}{3\ln 2}$ $\quad \bar{f} = \frac{1}{2}\int_1^3 x^2 \cdot 2^{x^3}\,dx$

B $\frac{2^{27}\cdot 27 - 2}{3}$ $\quad = \frac{1}{6}\int_1^3 2^{x^3}\cdot 3x^2\,dx$ $\quad u=x^3$, $du=3x^2dx$, $x=1\Rightarrow u=1$, $x=3\Rightarrow u=27$

C $\frac{2^{27}\ln 2 - 2\ln 2}{3}$ $\quad = \frac{1}{6}\int_1^{27} 2^u\,du$

D $\frac{2^{27}\cdot 27 - 2}{3\ln 2}$ $\quad = \frac{1}{6}\left.\frac{2^u}{\ln 2}\right]_1^{27} = \frac{1}{6}\frac{(2^{27}-2)}{\ln 2}$

E $\frac{2^{26} - 1}{3\ln 2}$ $\quad = \frac{1}{3\ln 2}(2^{26}-1)$

7. The marginal cost for a product is given by: $\frac{dc}{dq} = 0.003q^2 - 0.6q + 40$

If fixed costs are \$7,000 then the average cost of producing 100 units is:

A $9000

B $20

C $90

D $2000

E $10

$C = 0.001q^3 - 0.3q^2 + 40q + C_0$

$q = 0 \quad C = 7000 \Rightarrow C_0 = 7000$

$C = 0.001q^3 - 0.3q^2 + 40q + 7000$

$\bar{C} = 0.001q^2 - 0.3q + 40 + \frac{7000}{q}$

$q = 100 \Rightarrow \bar{C} = 10 - 30 + 40 + 70$

$= 90$

8. If the demand equation for a product is: $q = 300 - 10p$ and the supply equation is: $q = \frac{20p - 100}{3}$, then at market equilibrium, which of the following is false?

A $PS = \int_5^{20} \left(\frac{20p - 100}{3}\right) dp$

B $CS = \int_{20}^{30} (300 - 10p)\, dp$

C $PS = \int_0^{100} \left[20 - \left(\frac{3q + 100}{20}\right)\right] dq$

D $CS = \int_0^{100} \left[\left(\frac{300 - q}{10}\right) - 20\right] dq$

E $PS = \int_0^{100} \left(\frac{20p - 100}{3}\right) dp$

Equilibrium: $300 - 10p = \frac{20p - 100}{3}$

$900 - 30p = 20p - 100$

$1000 = 50p \Rightarrow p_0 = 20$

$\Rightarrow q_0 = 100$

$q = 300 - 10p \quad (q = 0 \Rightarrow p = 30)$

$\Rightarrow p = \frac{300 - q}{10}$

$q = \frac{20p - 100}{3} \quad (q = 0 \Rightarrow p = 5)$

$\Rightarrow p = \frac{3q + 100}{20}$

9. The area of the region enclosed by $x = y^3$, $y = 2$ and $x = -1$ is:

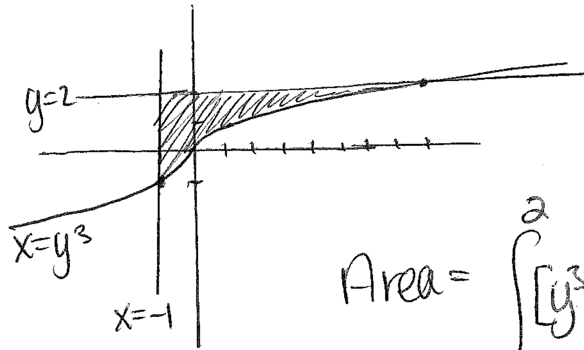
A 15/4

B 3/4

C 45/4

D 27/4

E 117/4



Area $= \int_{-1}^{2}[y^3-(-1)]dy = \int_{-1}^{2}[y^3+1]dy$

$= \left[\frac{y^4}{4}+y\right]_{-1}^{2} = \left[4+2-\frac{1}{4}+1\right]$

$= 6\frac{3}{4} = \frac{27}{4}.$

10. If $\frac{dy}{dx} = y^2 e^{2x}$ and $y = 2$ when $x=0$ then in general $y =$

A $\frac{-2}{e^{2x}-2}$

B $\frac{2}{3-2e^{2x}}$

C $\frac{y^3 e^{2x}+4}{6}$

D $\frac{8}{13-12e^{2x}}$

E $\frac{y^2 e^{2x}}{2}$

$\int \frac{dy}{y^2} = \int e^{2x}dx$

$-\frac{1}{y} = \frac{e^{2x}}{2}+C$

$\left.\begin{matrix} x=0 \\ y=2 \end{matrix}\right\} \Rightarrow -\frac{1}{2} = \frac{1}{2}+C \Rightarrow C=-1$

$-\frac{1}{y} = \frac{e^{2x}}{2}-1 = \frac{e^{2x}-2}{2}.$

$y = \frac{-2}{e^{2x}-2}.$

11. (a) Find the point(s) of intersection between the curves: $y = 2x^2$ and $y^2 = -4x$.

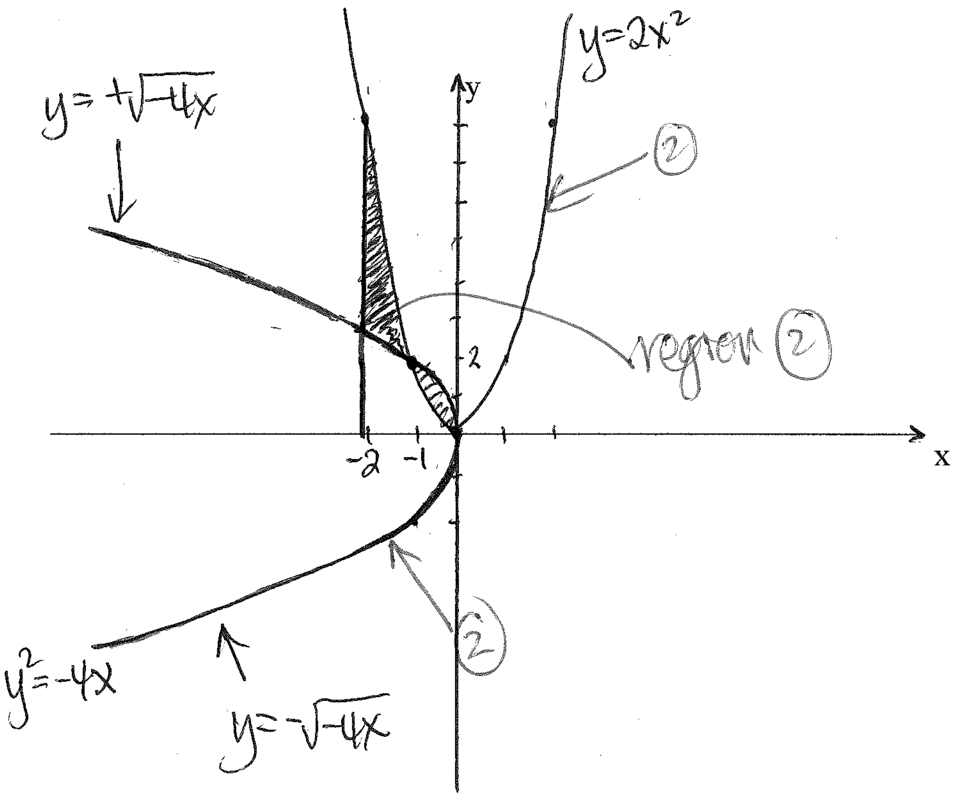
$\Rightarrow (2x^2)^2 = -4x \Rightarrow 4x^4 + 4x = 0$ 2

$\Rightarrow 4x(x^3+1) = 0 \Rightarrow x = 0$ or $x = -1$ 2

(5)

$\therefore$ intersection pts : $(0,0)$ $(-1,2)$ 1

(b) Sketch and shade the region between $y = 2x^2$ and $y^2 = -4x$ from $x = -2$ to $x = 0$.



(6)

(c) Write the area of the region in (b) in terms of definite integral(s), but do not evaluate the integral(s).

(4)

$$\text{Area} = \int_{-2}^{-1} \left[2x^2 - \sqrt{-4x}\right] dx + \int_{-1}^{0} \left[\sqrt{-4x} - 2x^2\right] dx$$

2 2

12. Evaluate the integral below. Show all steps clearly.

$$\int \frac{8x}{(x+1)(x-1)^2}\,dx = \int \left[\frac{-2}{x+1} + \frac{2}{x-1} + \frac{4}{(x-1)^2}\right] dx \quad 2$$

$$= -2\ln|x+1| + 2\ln|x-1| - \frac{4}{x-1} + C \quad 3$$

$$\text{or } 2\ln\left|\frac{x-1}{x+1}\right| - \frac{4}{x-1} + C$$

ie $$\frac{8x}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}. \quad 2$$

(15) $$= \frac{A(x-1)^2 + B(x+1)(x-1) + C(x+1)}{(x+1)(x-1)^2}. \quad 2$$

$$8x = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$$

$x=1 \Rightarrow 8 = 2C \Rightarrow C=4$ 2

$x=-1 \Rightarrow -8 = 4A \Rightarrow A=-2$. 2

$x=0 \Rightarrow 0 = -2 - B + 4 \Rightarrow B=2$ 2

13. Find the value in 10 years of a continuous annuity with payments at time t at the rate of $f(t) = 100t$ / yr and interest at 4% compounded continuously.

$$F.V = \int_0^{10} 100te^{.04(10-t)}\,dt. \quad (4)$$

let $f(t) = 100t \Rightarrow f'(t) = 100$

$g'(t) = e^{.4-.04t}$ (4)

$\Rightarrow g(t) = \dfrac{e^{.4-.04t}}{-.04}$

$$= \left[\frac{100te^{.4-.04t}}{-.04} - \int \frac{100e^{.4-.04t}}{-.04}\,dt\right]_0^{10} \quad (2)$$

$$= \left[-2500te^{.4-.04t} - 62500e^{.4-.04t}\right]_0^{10} \quad (2)$$

(15)

$$= \left[-25000e^{0} - 62500e^{0} + 0 + 62500e^{.4}\right]$$

$$\cong \left[-87500 + 93239.04\right]$$

$$\approx \$5739.04 \quad (3)$$

14. The number of bees in a hive at the beginning of May is 200. One month later, there are 500 bees in the hive. If the rate at which the nunber of bees is increasing is proportional to the number of bees at any time t, then

(a) How many bees there will be in the hive at the beginning of September?

Let B = # of bees in the hive at time t.

$\frac{dB}{dt} = kB \Rightarrow B = B_0 e^{kt}$ 2

2 $\left.\begin{array}{l} B_0 = 200 \\ t=1,\ B=500 \end{array}\right\} \Rightarrow 500 = 200e^{k} \Rightarrow e^{k} = 5/2$ 2

(9)

$\therefore B = 200\left(\frac{5}{2}\right)^{t}$ 1

$t=4 \Rightarrow B = 200\left(\frac{5}{2}\right)^{4} \approx 7812$ 2

$\therefore$ there will be approx 7812 bees at the beginning of September.

(b) When will there be approximately 2000 bees in the hive?

$B=2000 \Rightarrow 200\left(\frac{5}{2}\right)^{t} = 2000$ 2

$\Rightarrow \left(\frac{5}{2}\right)^{t} = 10.$

$\Rightarrow t\ln(5/2) = \ln 10.$ 2

(6)

$\Rightarrow t = \frac{\ln 10}{\ln 2.5} \approx 2.5.$ 2

$\therefore$ There will be approximately 2000 bees in the middle of July.