MAT 133Y1Y TERM TEST #3

THURSDAY, JULY 26, 2012 7:10 - 9:10 PM

FAMILY NAME: ______________________________

GIVEN NAMES: ______________________________

STUDENT NUMBER: ______________________________

TUTORIAL ROOM: ______________________________

Aids Allowed: Calculator with empty memory, to be supplied by the student. Absolutely no graphing calculators allowed.

Instructions: This test has 10 multiple choice questions worth 4 marks each and 5 written answer questions as shown below. For each multiple choice question, you may do your rough work in the test booklet, but you must record your answer by circling one of the letters A, B, C, D or E which appear on the front page of the test. A multiple choice question left blank, or having an incorrect answer circled, or having more than one answer circled, will be assigned a mark of 0. For the written answer solutions, present your solutions in the spaces provided. Use the back of the question pages for your rough work.

ANSWERS FOR MULTIPLE CHOICE **Circle the correct answer**					
1.	**A**	**B**	**C**	**D**	**E**
2.	**A**	**B**	**C**	**D**	**E**
3.	**A**	**B**	**C**	**D**	**E**
4.	**A**	**B**	**C**	**D**	**E**
5.	**A**	**B**	**C**	**D**	**E**
6.	**A**	**B**	**C**	**D**	**E**
7.	**A**	**B**	**C**	**D**	**E**
8.	**A**	**B**	**C**	**D**	**E**
9.	**A**	**B**	**C**	**D**	**E**
10.	**A**	**B**	**C**	**D**	**E**

GRADER'S REPORT	
Multiple Choice	/ 40
Question 11	/ 15
Question 12	/ 6
Question 13	/ 15
Question 14	/ 15
Question 15	/ 9
TOTAL	/100

1. Given: $f''(x) = 60x^2$ where $f(0) = 1$ and $f'(0) = 2$, then $f(1) =$

A 7

B 5

C 8

D 22

E 121

$f''(x) = 60x^2 \Rightarrow f'(x) = \int 60x^2\,dx$
$= 20x^3 + C_1$

$f'(0) = 2 \Rightarrow 0 + C_1 = 2 \Rightarrow C_1 = 2.$

$f(x) = \int (20x^3 + 2)\,dx = 5x^4 + 2x + C_2$

$f(0) = 1 \Rightarrow 0 + C_2 = 1 \Rightarrow C_2 = 1$

$\therefore f(x) = 5x^4 + 2x + 1$

$\Rightarrow f(1) = 8.$

2. $\lim_{n \to +\infty} \frac{3}{n} \{ \sqrt{1 + (3/n)} + \sqrt{1 + 2(3/n)} + \sqrt{1 + 3(3/n)} + \ldots\ldots + \sqrt{1 + n(3/n)} \}$

A 0

B $\frac{14}{3}$

C 3

D $\frac{16}{3}$

E $\frac{45}{2}$

$\Delta x = \frac{b-a}{n} = \frac{3}{n}$ $a = 1 \Rightarrow b = 4$

$f(x) = \sqrt{x}.$

$= \int_1^4 \sqrt{x}\,dx$

$= \left. \frac{x^{3/2}}{3/2} \right]_1^4 = \frac{2}{3}\left[4^{3/2} - 1\right]$

$= 14/3.$

3. Suppose the rate at which money in a savings account grows is directly proportional to the amount in the account at any time. If \$5000 is originally deposited into the account and there is \$7500 in 5 years, then the amount in the account in 10 years is closest to:

A \$11,201

B \$10,000

C \$11,250

D 12,450

E 12,000

let A = amt of money at time t

$$\frac{dA}{dt} = kA \Rightarrow A = A_0 e^{kt}$$

$A_0 = 5000$ when $t = 5$ $A = 7500$

$$\Rightarrow 7500 = 5000e^{5k} \Rightarrow e^{5k} = \frac{3}{2}$$

$$A = 5000(e^{5k})^{t/5} = 5000\left(\frac{3}{2}\right)^{t/5}$$

$$t = 10 \Rightarrow A = 5000\left(\frac{3}{2}\right)^2 = \frac{45000}{4}$$

$$= 11250$$

4. The average value of $f(x) = \dfrac{x^{4/3} + 1}{x}$ on the interval [1,8] is:

A $\dfrac{1}{8}$

B $\dfrac{1}{56}$

C $\dfrac{-25}{28}$

D $\dfrac{45}{28} + \dfrac{(\ln 8)}{7}$

E $\dfrac{45}{4} + \ln 8$

$$\bar{f} = \frac{1}{7}\int_1^8 \frac{x^{4/3}+1}{x}\,dx$$

$$= \frac{1}{7}\int_1^8 \left(x^{1/3} + \frac{1}{x}\right) dx$$

$$= \frac{1}{7}\left[\frac{x^{4/3}}{4/3} + \ln x\right]_1^8$$

$$= \frac{1}{7}\left[\frac{3}{4}(16) + \ln 8 - \frac{3}{4} - \ln 1\right]$$

$$= \frac{45}{28} + \frac{\ln 8}{7}$$

5. Given: MR $= \frac{dr}{dq} = 6q + 8$, the revenue received when 100 units are sold is:

$r = 3q^2 + 8q + C$

$q = 0 \Rightarrow r = 0 \Rightarrow C = 0$.

$q = 100$.

$\Rightarrow r = 3(100)^2 + 8(100)$

$= 30,800$.

A 608

B 3800

C 30,800

D 1100

E 1,040,000

6. The present value of a 15-year continuous annuity with payments at time t at the rate of f (t) = \$2000 /yr and interest at 3% compounded continuously is:

$$PV = \int_0^{15} 2000 e^{-0.03t} dt$$

$$= \left[\frac{2000}{-0.03} e^{-0.03t}\right]_0^{15}$$

$$= \frac{2000}{-0.03}\left[e^{-0.45} - e^0\right]$$

$$\approx 24158.12 .$$

A \$167,642.62

B \$23,875,87

C \$37,887.48

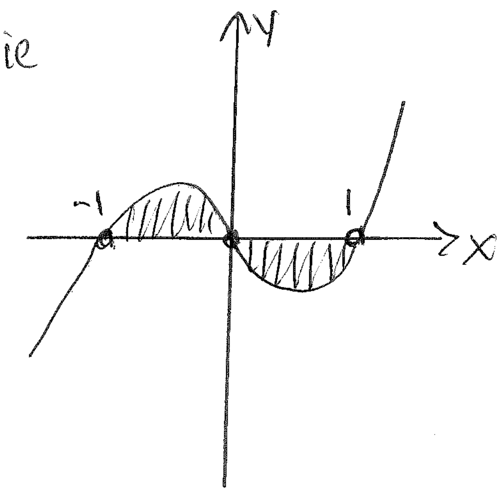
D \$24,158.12

E \$37,197.83

7. The area bounded by the curve $f(x) = x^3 - x$ and the x-axis is equal to:

$= x(x^2-1)$

ie



A $\frac{1}{2}$

B 4

C $\frac{1}{4}$

D 2

E 6

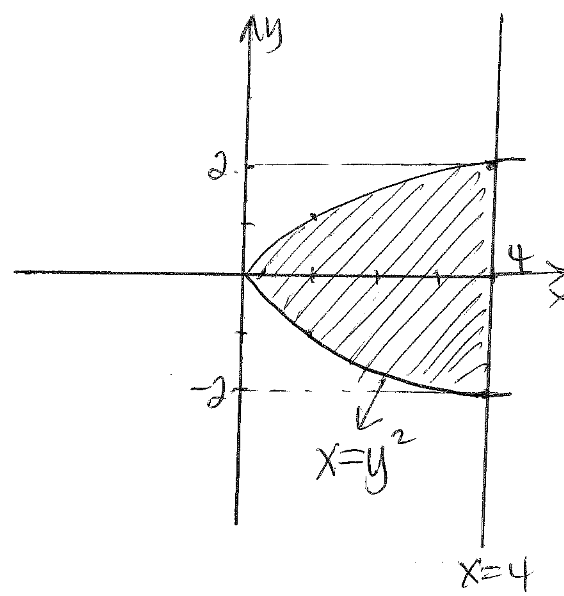
$$\text{Area} = 2\int_{-1}^{0}(x^3-x)\,dx$$

$$= 2\left[\frac{x^4}{4}-\frac{x^2}{2}\right]_{-1}^{0}$$

$$= 2\left[0-\frac{1}{4}+\frac{1}{2}\right]$$

$$= \frac{1}{2}$$

8. The area between the curves $x = y^2$ and $x = 4$ is equal to:



A $\int_0^4 \sqrt{x}\,dx$

B $\int_{-2}^{2} y^2\,dy$

C $\int_{-2}^{2} 2\sqrt{x}\,dx$

D $\int_{-2}^{2} (4-y^2)\,dy$

E $\int_0^4 y^2\,dy$

9. $\int_0^1 5^{(x^2)} x \, dx = \frac{1}{2}\int_0^1 5^{x^2}(2x)dx$.

$u = x^2$
$du = 2x\,dx$
$x=0 \Rightarrow u=0$
$x=1 \Rightarrow u=1$

A $\frac{5}{2\ln 5}$

$= \frac{1}{2}\int_0^1 5^u du$

B $\frac{5}{6}$

$= \frac{1}{2} \left. \frac{5^u}{\ln 5} \right]_0^1$

C $\frac{25}{2}$

D $\frac{2}{\ln 5}$

$= \frac{1}{2\ln 5}(5-1)$

E $\frac{5}{2}$

$= \frac{2}{\ln 5}$

10. If the demand equation for a product is: $p = 800 - 2q^2$ and the supply equation is: $p = 10q + 200$, then the Consumers' Surplus is given by: (at equilibrium)

A $\int_0^{15} (450 - 2q^2)\, dq$

B $\int_0^{350} (785 - 2q^2)\, dq$

C $\int_0^{15} (150 - 10q)\, dq$

D $\int_0^{20} (400 - 2q^2)\, dq$

E $\int_0^{15} (2q^2 - 450)\, dq$

Equilibrium:

$800 - 2q^2 = 10q + 200$.
$2q^2 + 10q - 600 = 0$.
$q^2 + 5q - 300 = 0$
$(q-15)(q+20) = 0$.
$q_0 = 15 \Rightarrow p_0 = 350$.

$CS = \int_0^{q_0} [\text{demand} - p_0]\, dq$.

$= \int_0^{15} [800 - 2q^2 - 350]\, dq$

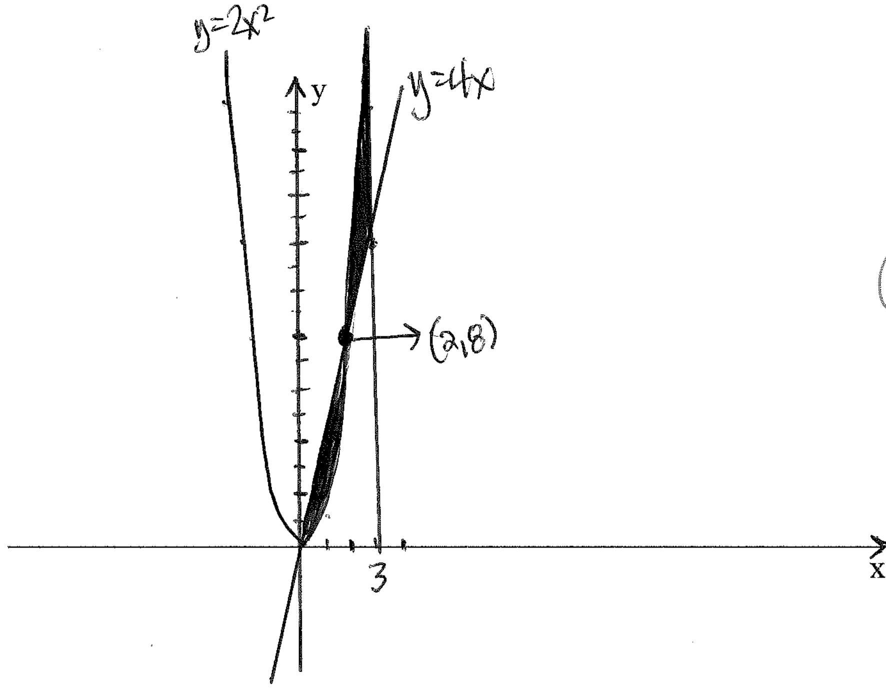
11. (a) Find the point(s) of intersection between the curves: $y = 2x^2$ and $y = 4x$.

(4)

$$2x^2 = 4x \Rightarrow 2x(x-2) = 0$$
$$\Rightarrow x = 0, 2 \; : \; \text{ie } (0,0)\ (2,8) \quad 4$$

(b) Sketch and shade the region between $y = 2x^2$ and $y = 4x$ from $x = 0$ to $x = 3$.

(4)



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(c) Write the area of the region in (b) in terms of definite integral(s), and evaluate the integral(s).

(7)

$$\text{Area} = \int_0^2 (4x - 2x^2)\,dx + \int_2^3 (2x^2 - 4x)\,dx \quad 2$$

$$= \left[2x^2 - \frac{2x^3}{3}\right]_0^2 + \left[\frac{2x^3}{3} - 2x^2\right]_2^3 \quad 2$$

$$= \left[8 - 16/3\right] + \left[(18-18) - \frac{16}{3} + 8\right]$$

$$= 16 - \frac{32}{3} = \frac{16}{3} \quad 3$$

12. Evaluate the integral below. Show all steps clearly.

$$\int \frac{\sqrt{\ln(x^3 + 3x)}\ (x^2 + 1)\ dx}{x^3 + 3x}$$

$$= \frac{1}{3}\int \frac{[\ln(x^3+3x)]^{1/2}(3x^2+3)}{(x^3+3x)}\,dx$$

$$u = \ln(x^3+3x)$$

$$du = \frac{1}{x^3+3x}(3x^2+3)dx$$

2

$$= \frac{1}{3}\int u^{1/2}\,du \quad 2$$

$$= \frac{1}{3}\,\frac{u^{3/2}}{3/2} + C \quad 1$$

$$= \frac{2}{9}\left[\ln(x^3+3x)\right]^{3/2} + C$$

1

(6)

13. Evaluate the integral below. Show all steps clearly.

$$\int_0^2 x^2 e^x \, dx$$

$f(x)=x^2 \Rightarrow f'(x)=2x$ (4)

$g'(x)=e^x \Rightarrow g(x)=e^x$

$= \left[x^2e^x - \int 2xe^x dx\right]_0^2$ (2)

$f(x)=2x \Rightarrow f'(x)=2$

$g'(x)=e^x \Rightarrow g(x)=e^x$ (4)

$= \left[x^2e^x - \left(2xe^x - \int 2e^x dx\right)\right]_0^2$ (2)

$= \left[x^2e^x - 2xe^x + 2e^x\right]_0^2$ (1)

$= \left[4e^2 - 4e^2 + 2e^2 - 0 + 0 - 2e^0\right]$

(15)

$= 2e^2 - 2$ (2)

14. (a) Evaluate the integral below. Show all steps clearly.

$$\int \frac{1}{x^2(x-1)}\,dx$$

$$\frac{1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} \quad 2$$

$$= \frac{Ax(x-1)+B(x-1)+Cx^2}{x^2(x-1)} \quad 2$$

$x=0 \Rightarrow 1 = -B \Rightarrow B = -1$

$x=1 \Rightarrow 1 = C$

$x=-1 \Rightarrow 1 = 2A - 2B + C$

$\Rightarrow 1 = 2A + 2 + 1$

$\Rightarrow A = -1 \quad 3$

$$= \int \left[\frac{-1}{x} - \frac{1}{x^2} + \frac{1}{x-1}\right] dx \quad 1$$

(10) $$= -\ln x + \frac{1}{x} + \ln|x-1| + C \quad 2$$

(b) By using the results of (a) evaluate the following integral:

$$\int_2^{+\infty} \frac{1}{x^2(x-1)}\,dx = \lim_{b\to+\infty} \int_2^b \frac{1}{x^2(x-1)}\,dx$$

$$= \lim_{b\to+\infty} \left[-\ln x + \frac{1}{x} + \ln|x-1|\right]_2^b \quad 1$$

$$= \lim_{b\to+\infty} \Big[\underbrace{-\ln b}_{-\infty} + \frac{1}{b} + \underbrace{\ln|b-1|}_{+\infty} + \ln 2 - \frac{1}{2} - \ln 1\Big] \quad 1$$

(5) $$= \lim_{b\to+\infty} \left[\ln\left|\frac{b-1}{b}\right| + \frac{1}{b} + \ln 2 - \frac{1}{2}\right] \quad 1$$

$$= \lim_{b\to+\infty} \Big[\ln\big|1-\underbrace{\tfrac{1}{b}}_{\to 0}\big| + \underbrace{\frac{1}{b}}_{\to 0} + \ln 2 - \frac{1}{2}\Big] \quad 1$$

$$= \ln 2 - \frac{1}{2} \quad 1$$

15. Given: $\frac{dy}{dx} = \frac{(2x^2 + 1)}{y^2(x-1)}$

If $y = 6$ when $x = 2$, then write y explicitly in terms of x.

$$\int y^2 dy = \int \frac{2x^2+1}{x-1} dx \quad (1)$$

$$\begin{array}{r} 2x+2 \\ x-1 \overline{\smash{)}\,2x^2+1} \\ \underline{2x^2-2x} \\ 2x+1 \\ \underline{2x-2} \\ 3 \end{array}$$

$$= \int \left[2x+2 + \frac{3}{x-1}\right] dx \quad (2)$$

$$\frac{y^3}{3} = x^2+2x+3\ln|x-1|+C \quad (2)$$

$$x=2, y=6 \Rightarrow 72 = 4+4+3\ln 1 + C$$

$$\Rightarrow C = 64 \quad (2)$$

(9)

$$\frac{y^3}{3} = x^2+2x+3\ln|x-1|+64$$

$$y^3 = 3x^2+6x+9\ln|x-1|+192$$

$$y = \sqrt[3]{3x^2+6x+9\ln|x-1|+192} \quad (2)$$