MAT 133Y1Y TERM TEST 3

Thursday, 24 July, 2014, 6:10 pm – 8:00 pm

Code 1

FAMILY NAME Solutions

GIVEN NAME(S) ______________________

STUDENT NO. ______________________

SIGNATURE ______________________

GRADER'S REPORT	
Question	Mark
MC/40	
B1/15	
B2/15	
B3/15	
B4/15	
TOTAL	

NOTE:

1. **Aids Allowed:** Calculator with empty memory, to be supplied by student. Absolutely no graphing calculators allowed.
2. **Instructions:** Fill in the information on this page and ensure that the test contains 10 pages.
3. This test has 10 multiple choice questions worth 4 marks each and 4 written-answer questions worth 15 marks each.
   For the **multiple choice questions** indicate your answers by circling the appropriate letters (**A**, **B**, **C**, **D**, or **E**) on **this page (page 1)**. A multiple choice question left blank on **this page**, or having an incorrect answer circled, or having more than one answer circled, will be assigned a mark of 0. For the **written-answer questions**, present your solutions in the spaces provided. Use the multiple choice question pages or the back of any of the pages for rough work, for any of the questions.

ANSWER BOX FOR PART A

Circle the correct answer.

1.	A	B	C	D	E
2.	A	B	C	D	E
3.	A	B	C	D	E
4.	A	B	C	D	E
5.	A	B	C	D	E
6.	A	B	C	D	E
7.	A	B	C	D	E
8.	A	B	C	D	E
9.	A	B	C	D	E
10.	A	B	C	D	E

PART A. Multiple Choice

1. *[4 marks]*

If differential approximation is used, then for $h \neq 0$ with $|h|$ small, $\sqrt{\frac{1}{16}+h}$ is approximately equal to

A $\frac{1}{4}+\frac{1}{2}h$

B $\frac{1}{4}+2h$

C $\frac{1}{4}+h$

D $\frac{1}{4}+2h^{-1}$

E $\frac{1}{4}+(2h)^{-1}$

With $f(x) = x^{\frac{1}{2}}$,

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$, $f(\frac{1}{16}) = \frac{1}{4}$,

and $f'(\frac{1}{16}) = 2$.

$$\sqrt{\tfrac{1}{16}+h} = f(\tfrac{1}{16}+h) \approx f(\tfrac{1}{16}) + f'(\tfrac{1}{16})\cdot h$$

$$= \frac{1}{4}+2h$$

2. *[4 marks]*

If $f''(x) = 6x$, $f'(1) = 4$, and $f(1) = 2$, then $f(2) =$

A 11

B 8

C 9

D 10

E 12

$f'(x) = \int f''(x)\,dx = \int 6x\,dx = 3x^2 + C_1$,

$4 = 3\cdot 1^2 + C_1$ so $C_1 = 1$,

$f'(x) = 3x^2+1$, and:

$f(x) = \int f'(x)\,dx = \int 3x^2+1\,dx = x^3+x+C_0$,

$2 = 1^3+1+C_0$, so $C_0 = 0$,

$f(x) = x^3+x$, and $f(2) = 2^3+2$.

3. *[4 marks]*

$$\int_{-1}^{0} x^3 \cdot 2^{(x^4)}\, dx =$$

A $-\frac{1}{4\ln 2}$

B $-\frac{1}{8\ln 2}$

C $-\frac{1}{2}$

D $-\frac{1}{4}$

E $\frac{1}{4\ln 2}$

Let $u = x^4$, so $\frac{du}{4} = x^3 dx$,

$u = 1$ when $x = -1$, $u = 0$ when $x = 0$,

and the integral equals

$$\int_1^0 2^u \frac{du}{4} = \int_1^0 e^{(\ln 2)u} \frac{du}{4}$$

$$= \frac{1}{4\ln 2}\left[e^{(\ln 2)u}\right]_1^0 = \frac{1}{4\ln 2}\left[2^u\right]_1^0 = \frac{2^0 - 2^1}{4\ln 2}$$

4. *[4 marks]*

$$\int \frac{4x^2+3}{2x-1}\, dx =$$

A $x^2 - x + C$

B $x^2 + x + 4\ln|2x-1| + C$

C $x^2 + x + 2\ln|2x-1| + C$

D $2x + 1 + \frac{4}{2x-1} + C$

E $x^2 + x + C$

[Side calculation:

$$\begin{array}{r|l} & 2x+1 \\ \hline 2x-1 & 4x^2+3 \\ & 4x^2-2x \\ \hline & 2x+3 \\ & 2x-1 \\ \hline & 4 \end{array}$$

]

$$\int \frac{4x^2+3}{2x-1}\, dx = \int 2x+1+\frac{4}{2x-1}\, dx = x^2+x+2\ln|2x-1| + C$$

5. *[4 marks]*

If $f(x) = \int_1^x \frac{dt}{1+t^2}$, $f'(3) =$ $\frac{1}{1+3^2}$ (fundamental theorem)

A $\frac{1}{5}$

B $\frac{1}{7}$

C $\frac{1}{10}$

D $\frac{1}{12}$

E $\frac{1}{8}$

6. *[4 marks]*

If a good has demand function $p = 15 - q^2$ and supply function $p = 2q$, then its producers' surplus is

A 12

B 8

C 15

D 6

E 9

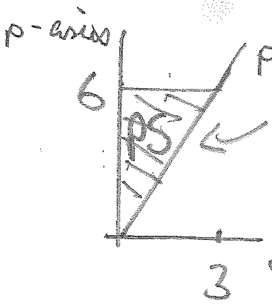
At equilibrium,

$2q = 15 - q^2$, $q^2 + 2q - 15 = 0$,

$(q+5)(q-3) = 0$, $q \neq 0$, so $q = 3$,

and $p = 2q = 6$.

Method ①



area $= \frac{1}{2} \cdot 3 \cdot 6$

Method ②

$\int_0^3 6 - 2q\, dq = \left[6q - q^2\right]_0^3$

$= 6 \cdot 3 - 3^2$

7. *[4 marks]*

If A and B are real numbers such that $\dfrac{A}{x+1}+\dfrac{B}{x-2}=\dfrac{2x-7}{(x+1)(x-2)}$ for all real x except $x=-1$ and $x=2$, then $A=$

A -1

B -3

C 2

D 1

E 3

A and B must satisfy

$A(x-2)+B(x+1)=2x-7$

for all x including $x=-1$:

$A(-1-2)+B\cdot 0=2(-1)-7$

8. *[4 marks]*

Beginning at a certain time ($t=0$, where t is given in years), cash flows continuously into an account at the rate $1000e^{.03t}$ dollars per year. If the account earns interest at the nominal annual rate of 7% compounded continuously and the cash flow stops at $t=10$ years, then its present value to the nearest dollar is

A \$9226

B \$8242

C \$8096

D \$7758

E \$8713

$\$\int_0^{10} 1000\, e^{.03t}\, e^{-.07t}\, dt$

$= \$\int_0^{10} 1000\, e^{-.04t}\, dt$

$= \$\left[-25000\, e^{-.04t}\right]_0^{10}$

$= \$\, 25000\left(1-e^{-.04\cdot 10}\right)$

9. *[4 marks]*

The average value of $f(x) = x(x+3)$ on the interval $[-3, 3]$ is

A 3

B 2

C 5

D 6

E 4

$$\frac{1}{3-(-3)} \int_{-3}^{3} x^2 + 3x \, dx$$

$$= \frac{1}{6} \left[\frac{x^3}{3} + \frac{3x^2}{2} \right]_{-3}^{3}$$

$$= \frac{1}{6} \left[\left(\frac{27}{3} + \cancel{\frac{27}{2}} \right) - \left(-\frac{27}{3} + \cancel{\frac{27}{2}} \right) \right]$$

$$= \frac{1}{6} \cdot 2 \cdot \frac{27}{3}$$

10. *[4 marks]*

$\int_4^{\infty} x^{-\frac{3}{2}} \, dx =$

A $\frac{1}{2}$

B 4

C 2

D 1

E ∞, that is, the integral diverges

$$\lim_{b \to \infty} \left[\int_4^b x^{-\frac{3}{2}} \, dx \right]$$

$$= \lim_{b \to \infty} \left[-2x^{-\frac{1}{2}} \right]_4^b$$

$$= \lim_{b \to \infty} \left[-\frac{2}{\sqrt{b}} + \frac{2}{\sqrt{4}} \right]$$

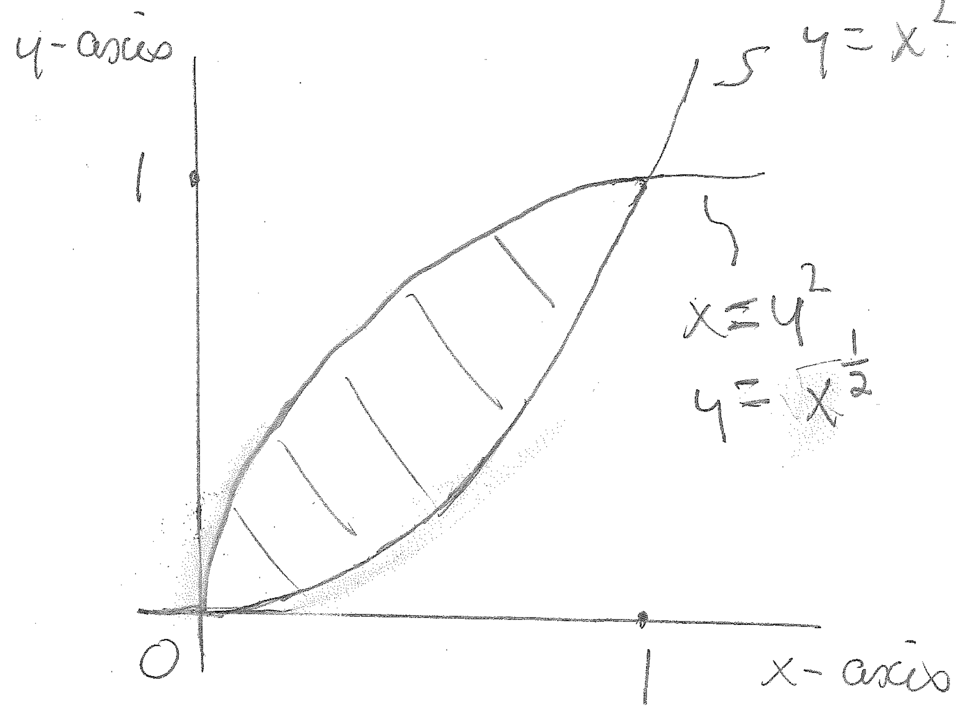
$$= 1$$

$(\sqrt{b} \to \infty \text{ as } b \to \infty)$

PART B. Written-Answer Questions
SHOW YOUR WORK.

B1. *[15 marks]*

Find the area bounded by the graphs of $y = x^2$ and $x = y^2$.



$$\int_0^1 x^{\frac{1}{2}} - x^2 \, dx = \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right]_0^1$$

$$= \frac{2}{3} - \frac{1}{3}$$

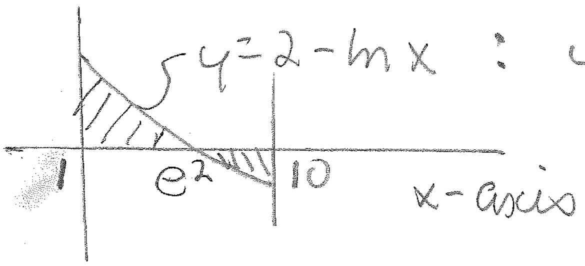
$$= \frac{1}{3}$$

B2. *[15 marks]*

Write the areas of the regions described below in terms of definite integrals without using absolute value signs **but do not evaluate the definite integrals.**

B2.(a) *[7 marks]*

The area between the graph of $y = 2 - \ln x$ and the x-axis from $x = 1$ to $x = 10$



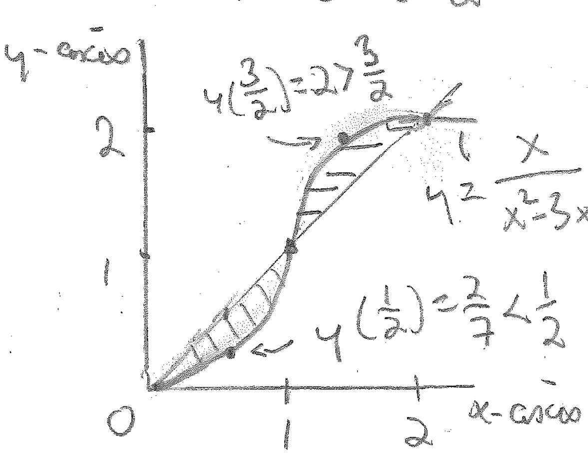
: $y=0$ iff $x=e^2$

$$\int_1^{e^2} 2-\ln x \, dx + \int_{e^2}^{10} (\ln x) - 2 \, dx$$

B2.(b) *[8 marks]*

The total area of the regions bounded by the line $y = x$ and the graph of $y = \dfrac{x}{x^2 - 3x + 3}$

Intersection at (x, y) iff x and $\dfrac{x}{x^2-3x+3}$ both equal y. Either $x=0$ or $x^2-3x+3=1$, $(x-1)(x-2)=0$, and $x=1$ or $x=2$.



$$\int_0^1 x - \frac{x}{x^2-3x+3} \, dx + \int_1^2 \frac{x}{x^2-3x+3} - x \, dx$$

B3. *[15 marks]*

Find the indefinite integrals:

B3.(a) *[8 marks]*

$\int x^2 e^{\frac{x}{3}}\, dx$ by parts (twice):

Let $u = x^2$, $dv = e^{\frac{x}{3}} dx$, so $du = 2x\,dx$, $v = 3e^{\frac{x}{3}}$,
and $\int x^2 e^{\frac{x}{3}} dx = 3x^2 e^{\frac{x}{3}} - 6\left(\int x e^{\frac{x}{3}} dx\right)$

$\int x e^{\frac{x}{3}} dx = \int x\,dv = xv - \int v\,dx$
$= 3xe^{\frac{x}{3}} - \int 3e^{\frac{x}{3}} dx = 3xe^{\frac{x}{3}} - 9e^{\frac{x}{3}}$

$\int x^2 e^{\frac{x}{3}} dx = 3x^2 e^{\frac{x}{3}} - 18xe^{\frac{x}{3}} + 54e^{\frac{x}{3}} + C$

$= 3(x^2 - 6x + 18)e^{\frac{x}{3}} + C$

B3.(b) *[7 marks]*

$$\int \frac{1-2x}{x^2(x-1)}\, dx$$

by partial fractions: We seek A, B, C so that
$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} = \frac{1-2x}{x^2(x-1)}$ for all $x \neq 0, x \neq 1$. Then

$Ax(x-1) + B(x-1) + Cx^2 = 1-2x$ for all x.
When $x = 0$, this implies $\underline{B=-1}$; when $x = 1$, $\underline{C=-1}$;
when $x = -1$, $2A - 2B + C = 3$, $2A + 2 + (-1) = 3$, and
$\underline{A=1}$. So $\int \frac{1-2x}{x^2(x-1)} dx = \int \frac{1}{x} - \frac{1}{x^2} - \frac{1}{x-1}\, dx$

$= \ln|x| + \frac{1}{x} - \ln|x-1| + C = \frac{1}{x} + \ln\left|\frac{x}{x-1}\right| + C$

B4. *[15 marks]*

Find $y(x)$ such that $xy' = y^2$ and $y(e) = 1$. Express y **explicitly** in terms of x. You may assume $x > 0$.

Separate variables: $\frac{dy}{y^2} = \frac{dx}{x}$

Integrate both sides:

$$-\frac{1}{y} = \ln x + C$$ (general solution)

$(x > 0)$

C must ensure $y(e) = 1$:

$-\frac{1}{1} = \ln e + C$ and $C = -2$

So $-\frac{1}{y} = (\ln x) - 2$ (particular solution)

Then $\frac{1}{y} = 2 - \ln x$ so

$$y(x) = \frac{1}{2 - \ln x}$$ (particular solution written explicitly)