MAT 133Y1Y TERM TEST #3

THURSDAY, JULY 23, 2009 7:30 - 9:30 PM

FAMILY NAME: ______________________________

GIVEN NAMES: ______________________________

STUDENT NUMBER: ______________________________

TUTORIAL ROOM: ______________________________

Aids Allowed: Calculator with empty memory, to be supplied by the student. Absolutely no graphing calculators allowed.
Instructions: This test has 10 multiple choice questions worth 4 marks each and 5 written answer questions worth a total of 60 marks. For each multiple choice question, you may do your rough work in the test booklet, but you must record your answer by circling one of the letters A, B, C, D or E which appear on the front page of the test. A multiple choice question left blank, or having an incorrect answer circled, or having more than one answer circled, will be assigned a mark of 0. For the written answer solutions, present your solutions in the spaces provided. Use the back of the question pages for your rough work.

GRADER'S REPORT	
Multiple Choice	/ 40
Question 11	/ 15
Question 12	/ 15
Question 13	/ 16
Question 14	/ 14
TOTAL	/100

ANSWERS FOR MULTIPLE CHOICE Circle the correct answer					
1.	A	B	C	D	E
2.	A	B	C	D	E
3.	A	B	C	D	E
4.	A	B	C	D	E
5.	A	B	C	D	E
6.	A	B	C	D	E
7.	A	B	C	D	E
8.	A	B	C	D	E
9.	A	B	C	D	E
10.	A	B	C	D	E

1. If $f''(x) = \frac{12}{x^3}$, $f'(1) = 0$ and $f(1) = 20$, then $f(2) =$

A 23

B 25

C 9/2

D 3

E 21

$f''(x) = \frac{12}{x^3} \Rightarrow f'(x) = \frac{-6}{x^2} + C_1$

$f'(1) = 0 \Rightarrow \frac{-6}{1} + C_1 = 0 \Rightarrow C_1 = 6$

$f'(x) = \frac{-6}{x^2} + 6 \Rightarrow f(x) = \frac{6}{x} + 6x + C_2$

$f(1) = 20 \Rightarrow 6 + 6 + C_2 = 20 \Rightarrow C_2 = 8$

$\therefore f(x) = \frac{6}{x} + 6x + 8 \Rightarrow f(2) = 3 + 12 + 8$
$= 23$

2. $\lim_{n \to +\infty} \frac{2}{n} \{ [1 + (2/n)]^4 + [1 + 2(2/n)]^4 + \ldots\ldots\ldots\ldots + [1 + n(2/n)]^4 \} =$

A $\int_1^3 (1 + x)^4 \, dx$

B $\int_0^2 (1 + x^4) \, dx$

C $\int_0^2 x^4 \, dx$

D $\int_1^3 x^4 \, dx$

E $\int_1^3 (1 + x^4) \, dx$

$f(x) = x^4 \quad a = 1$

$\Delta x = \frac{b-a}{n} = \frac{2}{n} \Rightarrow b = 3$

or

$f(x) = (1+x)^4 \quad a = 0$

$\Delta x = \frac{2}{n} \Rightarrow b = 2$

3. If marginal cost is given by $\frac{dc}{dq} = 0.4q + 2$ (where cost is in dollars), then how much does it cost to increase production from 40 units to 50 units ?

$c = \int (0.4q + 2)\,dq$

$= 0.2q^2 + 2q + C_0$

change in cost from 40 → 50

$\int_{40}^{50} (0.4q + 2)\,dq = \left[0.2q^2 + 2q\right]_{40}^{50}$

$= [(500 + 100) - (320 + 80)]$

$= 200$

A $400

B $22

C $600

D $200

E $4

4. $\frac{d}{dx} \int_1^{x^2} f(t)\,dt =$ $\frac{d}{dx}\left[F(t)\right]_1^{x^2}$ where $F'(t) = f(t)$

$= \frac{d}{dx}\left[F(x^2) - F(1)\right]$

$= F'(x^2)(2x) - 0$

$= f(x^2)(2x)$

A $f(x^2)$

B $f(2x) - f(0)$

C $f(2x)$

D $2x\, f(x^2)$

E $2x\, f(x^2) - f(0)$

5. $\int_{-1}^{0} 5^{(x^3-3x)}(x^2-1)\,dx = \frac{1}{3}\int_{-1}^{0} 5^{(x^3-3x)}(3x^2-3)\,dx$

$u = x^3-3x$

$du = (3x^2-3)dx$

$x=-1 \Rightarrow u=2$

$x=0 \Rightarrow u=0$

$= \frac{1}{3}\int_{2}^{0} 5^u\,du$

$= \frac{1}{3}\,\frac{5^u}{\ln 5}\Big]_2^0$

$= \frac{1}{3}\left[\frac{1}{\ln 5} - \frac{5^2}{\ln 5}\right]$

$= \frac{-8}{\ln 5}$

A $\frac{-50}{3}$

B $\frac{-24}{\ln 5}$

C $\frac{-8}{\ln 5}$

D $\frac{8}{\ln 5}$

E -8

6. $\int \frac{6t^3\,dt}{t-1} =$

$$\begin{array}{r} 6t^2+6t+6 \\ t-1\,\overline{)\,6t^3} \\ \underline{6t^3-6t^2} \\ 6t^2 \\ \underline{6t^2-6t} \\ 6t \\ \underline{6t-6} \\ 6 \end{array}$$

$= \int\left[6t^2+6t+6+\frac{6}{t-1}\right]dt$

$= 2t^3+3t^2+6t+6\ln|t-1|+C$

A $2t^3 + 3t^2 + 6t + C$

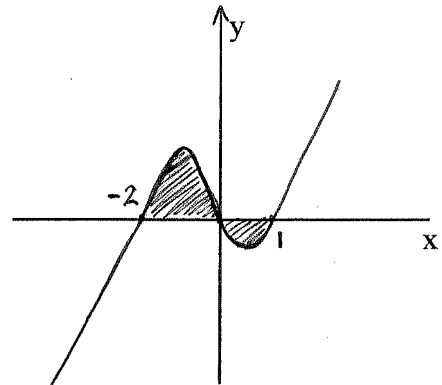
B $2t^3 + 3t^2 + 6t + 6\ln|t-1| + C$

C $3t^2 - 2t^3 + C$

D $6t^2 + 6t + 6 + \frac{6}{t-1} + C$

E $\frac{4t^3}{t^2-2t} + C$

7. The area of the shaded regions shown to the right is equal to



$y = x^3 + x^2 - 2x$ $= x(x^2+x-2)$
$= x(x+2)(x-1)$

A 9/4

B 37/12

C 19/12

D 15/4

E 13/6

$$\int_{-2}^{0}[x^3+x^2-2x]\,dx - \int_{0}^{1}[x^3+x^2-2x]\,dx$$

$$= \left[\frac{x^4}{4}+\frac{x^3}{3}-x^2\right]_{-2}^{0} - \left[\frac{x^4}{4}+\frac{x^3}{3}-x^2\right]_{0}^{1}$$

$$= \left[0-\left(4-\frac{8}{3}-4\right)\right]-\left[\left(\frac{1}{4}+\frac{1}{3}-1\right)-0\right] = \frac{37}{12}$$

8. If the demand function for a product is $p = \dfrac{1000}{q+20} - 10$

and the supply function is $q = 4p - 10$

then the Consumers' Surplus at market equilibrium is given by

A $\displaystyle\int_0^{30} \left(20 - \frac{1000}{q+20}\right) dq$

B $\displaystyle\int_0^{10} \left(\frac{1000}{q+20} - 40\right) dq$

C $\displaystyle\int_0^{30} \left(\frac{1000}{q+20} - 20\right) dq$

D $\displaystyle\int_{10}^{30} \left(\frac{1000}{p+10} - 20\right) dp$

E $\displaystyle\int_0^{30} \left(10 - \frac{q+10}{4}\right) dq$

Equlibrium:

$p = \dfrac{1000}{4p-10+20} - 10$

$p+10 = \dfrac{1000}{4p+10}$

$(p+10)(4p+10) = 1000$

$4p^2 + 50p - 900 = 0$

$2p^2 + 25p - 450 = 0$

$(2p+45)(p-10) = 0$

$\Rightarrow p_0 = 10 \Rightarrow q_0 = 30$

$CS = \displaystyle\int_0^{30}\left[\frac{1000}{q+20} - 10 - 10\right] dq$

9. Suppose that the rate at which a company's daily sales fall is proportional to the daily sales t days after an advertising campaign has ended . If the daily sales are $2000 when the campaign ends and 10 days later they are $1000, then the daily sales in 25 days will be closest to:

S = Sales t days after campaign

$\frac{dS}{dt} = KS \Rightarrow S = S_0 e^{Kt}$

A $300

B $425

C $325

D $350

E $375

$t=0 \Rightarrow S_0 = 2000$

$t=10 \Rightarrow S = 1000$

$\Rightarrow 1000 = 2000 e^{10K}$

$e^{10K} = \frac{1}{2}$

$S = 2000\left(\frac{1}{2}\right)^{t/10}$

$t=25 \Rightarrow S = 2000\left(\frac{1}{2}\right)^{5/2} \approx 354$

10. The average value of $f(x) = \dfrac{3e^{\sqrt{x}}}{2\sqrt{x}}$ on the interval [1,4] is:

A $\dfrac{e^2}{2} - e$

B $e^2 - e$

C $\dfrac{e^2}{12} - \dfrac{2e}{3}$

D $6e^2 - 6e$

E $2e^2 - 2e$

$\bar{f} = \frac{1}{3}\int_1^4 \frac{3e^{\sqrt{x}}}{2\sqrt{x}}\,dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$x=1 \Rightarrow u=1$

$x=4 \Rightarrow u=2$

$= \int_1^2 e^u\,du$

$= e^u\Big]_1^2$

$= e^2 - e$

11. Find the future value of a continuous annuity at an annual rate of 3% compounded continuously for 15 years, if the payment (in dollars) at time t is at the annual rate of 40t.

$$FV = \int_0^{15} 40t\, e^{0.03(15-t)}\,dt \quad ③$$

$$f(t) = 40t \Rightarrow f'(t) = 40$$
$$g'(t) = e^{0.45-0.03t}$$
$$\Rightarrow g(t) = \frac{e^{0.45-0.03t}}{-0.03} \quad ④$$

$$FV = \left[\frac{40te^{0.45-0.03t}}{-0.03} - \int \frac{40e^{0.45-0.03t}}{-0.03}\,dt\right]_0^{15} \quad ③$$

$$= \left[\frac{-4000te^{0.45-0.03t}}{3} - \frac{40e^{0.45-0.03t}}{0.0009}\right]_0^{15} \quad ②$$

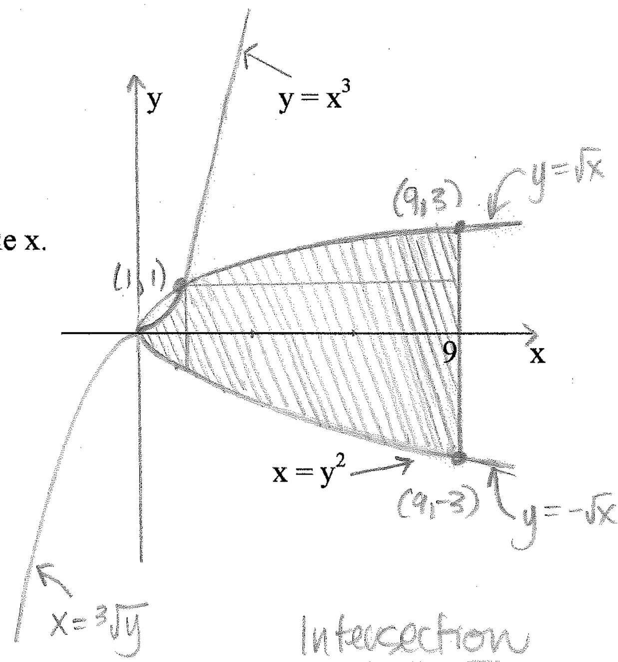
(15)

$$= \left[-20000\,e^{0} - \frac{400000\,e^{0}}{9} + 0 + \frac{400000\,e^{0.45}}{9}\right] \quad ①$$

$$= -20000 - \frac{400000}{9} + \frac{400000\,e^{0.45}}{9}$$

$$\approx \$5,258.32 \quad ②$$

12. Write the area of the shaded region:



(a) In terms of definite integrals in the variable x.
Do not evaluate the integrals.

Area Between Curves

$$= \int_0^1 [x^3-(-\sqrt{x})]\,dx \quad 4$$

(8) $$+ \int_1^9 [\sqrt{x}-(-\sqrt{x})]\,dx \quad 4$$

$$= \int_0^1 (x^3+\sqrt{x})\,dx + \int_1^9 2\sqrt{x}\,dx$$

Intersection

$$x^3 = x^2$$

$$x^3 - x^2 = 0$$

$$x^2(x-1) = 0$$

$$x = 0, 1$$

$$(0,0) \quad (1,1)$$

OR $$\int_0^9 [\sqrt{x}-(-\sqrt{x})]\,dx - \int_0^1 [\sqrt{x}-x^3]\,dx$$

$$= \int_0^9 2\sqrt{x}\,dx - \int_0^1 \sqrt{x}-x^3\,dx$$

(b) In terms of definite integrals in the variable y.
Do not evaluate the integrals.

Area Between Curves

(8) $$= \int_{-3}^{3} [9-y^2]\,dy - \int_0^1 [\sqrt[3]{y}-y^2]\,dy \quad 4 \quad 4$$

or $$\int_{-3}^{0} [9-y^2]\,dy + \int_0^1 [9-\sqrt[3]{y}]\,dy + \int_1^3 [9-y^2]\,dy$$

13. Write the area of the region between the curve $f(x) = \dfrac{1}{x^2(x+1)}$ and the x-axis, to the right of the line x= 1, as a definite integral and evaluate the integral.

$$\text{Area} = \int_1^{+\infty} \frac{1}{x^2(x+1)}\,dx \quad (2)$$

since $x^2(x+1) > 0$ on $[1, +\infty)$

$$= \lim_{b \to +\infty} \int_1^b \left[\frac{-1}{x} + \frac{1}{x^2} + \frac{1}{x+1} \right] dx \quad (2)$$

$$= \lim_{b \to +\infty} \left[-\ln x - \frac{1}{x} + \ln(x+1) \right]_1^b \quad (2)$$

(15)

$$= \lim_{b \to +\infty} \left[-\ln b - \frac{1}{b} + \ln(b+1) + \ln 1 + 1 - \ln 2 \right] \quad (1)$$

$$= \lim_{b \to +\infty} \left[\underbrace{\ln\left(\frac{b+1}{b}\right)}_{\to 0} - \underbrace{\frac{1}{b}}_{\to 0} + 1 - \ln 2 \right]. \quad (2)$$

by L'hopital's $\frac{b+1}{b} = \frac{1}{1}$ as $b \to +\infty$

$$= 1 - \ln 2 \quad (1)$$

ie: $$\frac{1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} = \frac{Ax(x+1) + B(x+1) + Cx^2}{x^2(x+1)} \quad (2)$$

$x = 0 \Rightarrow 1 = B \quad (1)$

$x = -1 \Rightarrow 1 = C \quad (1)$

$x = 1 \Rightarrow 1 = 2A + 2B + C \Rightarrow 1 = 2A + 2 + 1$

$\Rightarrow A = -1 \quad (1)$

14. Suppose that a chain of auto service stations has found that the relationship between the price (p) of an oil change in dollars and the station's monthly revenue (r) in 1000's of dollars is given by:

$$\frac{dr}{dp} = \frac{-r}{2(p+5)}$$

When the price of an oil change is \$20, the monthly revenue is \$8000 (i.e. r = 8) Find the monthly revenue when the price of an oil change is \$59.

$$\int \frac{-2}{r}\,dr = \int \frac{dp}{p+5} \qquad 2$$

$$-2\ln r = \ln(p+5) + C \qquad 2$$

$$\ln r^{-2} = \ln(p+5) + C \qquad 1$$

(14)

$$\frac{1}{r^2} = (p+5)e^c \qquad 3$$

$$p=20, r=8 \Rightarrow \frac{1}{64} = (25)e^c$$

$$\Rightarrow e^c = \frac{1}{1600} \qquad 2$$

$$\therefore \frac{1}{r^2} = \frac{(p+5)}{1600} \qquad 1$$

$$p=59 \Rightarrow \frac{1}{r^2} = \frac{64}{1600}$$

$$\Rightarrow r^2 = 25$$

$$\Rightarrow r = 5 \qquad 2$$

∴ monthly revenue is \$5000 when the price is \$59. 1