Problem Sets

Page numbers below refer to Kolman and Beck, 2^{nd} edition. It is recommended that you also do the odd numbered problems, whose answers appear near the end of the book.

problem set 1: pg. 21 #6.(b). Additional instructions: (i). Solve for x, y, and z in terms of w. (ii). Solve for x, z, and w in terms of y. pg. 21 #9.(a), pg. 28 #6.(c), #8.(b), pg. 42 #5.(d), #6.(b).

problem set 2: pg. 57 #2, #4, pg. 58 #6, #8, pg. 59 #10. Put these problems in canonical and in standard form.

problem set 3: pg. 82 #14, pg. 83 #16. In the preceding questions, replace instructions (b) and (c) with "draw the line $z = c^{\mathrm{T}}x = k$, where k is the optimal value of z." pg. 91 #4, #8, #12, pg. 100 #6, #8.

(A warning: the answers at the bottom of pg. 419 for #7.(a) and #7.(c) are incorrect. The correct answer for #7.(a) is: "Not even a solution". The correct answer to #7.(c) is the answer pg. 417 has for #7.(a).)

Supplementary problems:

1. Prove that the set of optimal solutions of any linear programming problem is convex.

2. Prove that the set of objective values which a linear programming problem attains over its feasible region is convex.

3. Give an example of a convex set in $\mathbb{R}^2$, which is not a line segment, and which has $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ as its only extreme points.

4. Find all extreme points (in $\mathbb{R}^3$) of

$$\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \text{ s.t. } x_1 - x_2 + x_3 = -1, 3x_1 - 2x_2 + 4x_3 = 2, x_1 + x_2 + 3x_3 = 9, x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \}.$$

problem set 4: pg. 120 #6 (solve the problem which this tableau represents), #8 (insert row labels and solve the problem), pg. 121 #14, #16, #19, pg. 122 #22, #23, pg. 131 #6.

problem set 5: pg. 150 #2, pg. 152 #10, #20.

Supplementary problems:

1. Maximize $z = -x_1 + 5x_2 + 3x_3$ subject to the constraints $\begin{array}{rcrcrcr} 2x_1 & + & x_2 & + & x_3 & = & 5 \\ 3x_1 & + & 2x_2 & & & \geq & 6 \\ 4x_1 & + & 3x_2 & - & x_3 & \leq & 7 \end{array}$, $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$.

2. Maximize $z = -9x_1 \qquad - 4x_3$ subject to the constraints $\begin{array}{rcrcrcr} 9x_1 & + & x_2 & + & x_3 & \geq & 27 \\ 3x_1 & + & x_2 & + & 2x_3 & = & 9 \end{array}$, $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$.

problem set 6: pg. 165 #2, #4, pg. 166 #6, pg. 183 #9. Modify this problem by replacing "x_1, x_2, x_3 unrestricted" with "$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$". pg. 184 #11.

Supplementary problem (pertaining to §3.3):

Consider the standard problem: Maximize $z = -76x_1 + 13x_2 - 11x_3 - 27x_4 + 2x_5$ subject to the constraints

$$\begin{array}{rcrcrcrcrcr} 11x_1 & + & x_2 & - & x_3 & - & 7x_4 & + & x_5 & \leq & 3 \\ -15x_1 & - & x_2 & + & 2x_3 & + & 23x_4 & - & 3x_5 & \leq & 2 \\ -7x_1 & + & 2x_2 & - & 2x_3 & - & 9x_4 & + & x_5 & \leq & 4 \end{array}, x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0.$$

a) Insert slack variables x_6, x_7, x_8 for the $1^{\text{st}}, 2^{\text{nd}}$, and 3^{rd} constraints respectively, and write the initial simplex tableau for the problem.

b) The final simplex tableau for the problem has basic variables x_5, x_3, x_2 (in that order). <u>Without using the simplex method</u>, write the final simplex tableau for the problem.

c) Let $w_i (i = 1, 2, 3)$ be the dual variable associated with the i^{th} primal constraint. What is the optimal solution of the dual problem?

problem set 7: pg. 214 #2, pg. 215 #4, #8, pg. 223 #2, #4, pg. 224 #6, #8.