pg. 150 2.

phase 1, Tableau ⓪ (slack, artificial)

	x_1	x_2	x_3	x_4	x_5	x_6	y_1	y_2	
x_5	1	3	-1	1	1	0	0	0	5
y_1	1	7	1	0	0	-1	1	0	4
y_2	4	2	0	1	0	0	0	1	3
	0	0	0	0	0	0	1	1	0

phase 1, Tableau ① (θ-ratios)

	x_1	x_2	x_3	x_4	x_5	x_6	y_1	y_2		
x_5	1	3	-1	1	1	0	0	0	5	$\frac{5}{3}$
y_1	1	(7)	1	0	0	-1	1	0	4	$\frac{4}{7}$
y_2	4	2	0	1	0	0	0	1	3	$\frac{3}{2}$
	-5	-9	-1	-1	0	1	0	0	-7	

phase 1, Tableau ② (θ-ratios)

	x_1	x_2	x_3	x_4	x_5	x_6	y_1	y_2		
x_5	$\frac{4}{7}$	0	$-\frac{12}{7}$	1	1	$\frac{3}{7}$	$-\frac{3}{7}$	0	$\frac{23}{7}$	$\frac{23}{4}$
x_2	$\frac{1}{7}$	1	$\frac{1}{7}$	0	0	$-\frac{1}{7}$	$\frac{1}{7}$	0	$\frac{4}{7}$	$\frac{4}{1}$
y_2	$(\frac{26}{7})$	0	$-\frac{2}{7}$	1	0	$\frac{2}{7}$	$-\frac{2}{7}$	1	$\frac{13}{7}$	$\frac{13}{26}$
	$-\frac{26}{7}$	0	$\frac{2}{7}$	-1	0	$-\frac{2}{7}$	$\frac{9}{7}$	0	$-\frac{13}{7}$	

phase 1, Tableau ③

	x_1	x_2	x_3	x_4	x_5	x_6	y_1	y_2	
x_5	0	0	$-\frac{18}{13}$	$\frac{11}{13}$	1	$\frac{5}{13}$	$-\frac{5}{13}$	$-\frac{2}{13}$	3
x_2	0	1	$\frac{2}{13}$	$-\frac{1}{26}$	0	$-\frac{2}{13}$	$\frac{2}{13}$	$-\frac{1}{26}$	$\frac{1}{2}$
x_1	1	0	$-\frac{1}{13}$	$\frac{7}{26}$	0	$\frac{1}{13}$	$-\frac{1}{13}$	$\frac{7}{26}$	$\frac{1}{2}$
	0	0	0	0	0	0	1	1	0

Pg 150 2. (cont'd)

phase 2, Tableau ①

	x_1	x_2	x_3	x_4	x_5	x_6	
x_5	0	0	$-\frac{18}{13}$	$\frac{11}{13}$	1	$\frac{5}{13}$	3
x_2	0	1	$\frac{2}{13}$	$-\frac{1}{26}$	0	$-\frac{2}{13}$	$\frac{1}{2}$
x_1	1	0	$-\frac{1}{13}$	$\frac{7}{26}$	0	$\frac{1}{13}$	$\frac{1}{2}$
	-1	-2	0	-1	0	0	0

phase 2, Tableau ②

θ-ratios

	x_1	x_2	x_3	x_4	x_5	x_6		
x_5	0	0	$-\frac{18}{13}$	$\frac{11}{13}$	1	$\frac{5}{13}$	3	$\frac{39}{11}$
x_2	0	1	$\frac{2}{13}$	$-\frac{1}{26}$	0	$-\frac{2}{13}$	$\frac{1}{2}$	
x_1	1	0	$-\frac{1}{13}$	$\frac{7}{26}$	0	$\frac{1}{13}$	$\frac{1}{2}$	$\frac{26}{14}$
	0	0	$\frac{3}{13}$	$-\frac{21}{26}$	0	$-\frac{3}{13}$	$\frac{3}{2}$	

phase 2, Tableau ③

	x_1	x_2	x_3	x_4	x_5	x_6	
x_5	$-\frac{22}{7}$	0	$-\frac{8}{7}$	0	1	$\frac{1}{7}$	$\frac{10}{7}$
x_2	$\frac{1}{7}$	1	$\frac{1}{7}$	0	0	$-\frac{1}{7}$	$\frac{4}{7}$
x_4	$\frac{26}{7}$	0	$-\frac{2}{7}$	1	0	$\frac{2}{7}$	$\frac{13}{7}$
	3	0	0	0	0	0	3

Z is maximal at $[x_1\ x_2\ x_3\ x_4\ x_5\ x_6]^T$

$= [0\ \frac{4}{7}\ 0\ \frac{13}{7}\ \frac{10}{7}\ 0]^T$ where its value is 3.

pg 152 10.

phase 1, Tableau ①

	x	y	x_1	x_2	x_3	y_1	y_2	y_3	
y_1	2	3	-1	0	0	1	0	0	18
y_2	1	3	0	-1	0	0	1	0	12
y_3	4	3	0	0	-1	0	0	1	24
	0	0	0	0	0	1	1	1	0

slacks; artificial

phase 1, Tableau ①

	x	y	x_1	x_2	x_3	y_1	y_2	y_3		θ-ratios
y_1	2	3	-1	0	0	1	0	0	18	6
y_2	1	(3)	0	-1	0	0	1	0	12	4
y_3	4	3	0	0	-1	0	0	1	24	8
	-7	-9	1	1	1	0	0	0	-54	

phase 1, Tableau ②

	x	y	x_1	x_2	x_3	y_1	y_2	y_3		θ-ratios
y_1	1	0	-1	1	0	1	-1	0	6	6
y	$\frac{1}{3}$	1	0	$-\frac{1}{3}$	0	0	$\frac{1}{3}$	0	4	12
y_3	(3)	0	0	1	-1	0	-1	1	12	4
	-4	0	1	-2	1	0	3	0	-18	

phase 1, Tableau ③

	x	y	x_1	x_2	x_3	y_1	y_2	y_3		θ-ratios
y_1	0	0	-1	(3)	$\frac{1}{3}$	1	$-\frac{2}{3}$	$-\frac{1}{3}$	2	3
y	0	1	0	$-\frac{4}{9}$	$-\frac{1}{9}$	0	$\frac{4}{9}$	$-\frac{1}{9}$	$\frac{8}{3}$	
x	1	0	0	$\frac{1}{3}$	$-\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{1}{3}$	4	12
	0	0	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$\frac{5}{3}$	$\frac{4}{3}$	-2	

pg. 152 10. (cont'd)

phase 1, Tableau (4)

	x	y	x_1	x_2	x_3	y_1	y_2	y_3	
x_2	0	0	$-\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{3}{2}$	-1	$-\frac{1}{2}$	3
y	0	1	$-\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	0	$-\frac{1}{3}$	4
x	1	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	3
	0	0	0	0	0	1	1	1	0

phase 2, Tableau (0)

	x	y	x_1	x_2	x_3	
x_2	0	0	$-\frac{3}{2}$	1	$\frac{1}{2}$	3
y	0	1	$-\frac{2}{3}$	0	$\frac{1}{3}$	4
x	1	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	3
	20	25	0	0	0	0

phase 2, Tableau (1)

	x	y	x_1	x_2	x_3	
x_2	0	0	$-\frac{3}{2}$	1	$\frac{1}{2}$	3
y	0	1	$-\frac{2}{3}$	0	$\frac{1}{3}$	4
x	1	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	3
	0	0	$\frac{20}{3}$	0	$\frac{5}{3}$	-160

these are positive because the problem is to maximize $-20x-25y$

Optimal solution: $[x\ y\ x_1\ x_2\ x_3]^T$
$= [3\ 4\ 0\ 3\ 0]^T$ with $z = 20x + 25y$
taking the minimal value of 160.

pg. 152 20.

phase 1, Tableau (0)

	x_1	x_2	x_3	y_1	y_2	
y_1	-4	2	6	1	0	4
y_2	6	9	12	0	1	3
	0	0	0	1	1	0

artificial

phase 1, Tableau (1) θ-ratios

	x_1	x_2	x_3	y_1	y_2		
y_1	-4	2	6	1	0	4	$\frac{4}{6}$
y_2	6	9	(12)	0	1	3	$\frac{3}{12}$
	-2	-11	-18	0	0	-7	

Pg. 152 20. (cont'd)

phase 1, Tableau (2)

	x_1	x_2	x_3	y_1	y_2	
y_1	$-\frac{7}{2}$	$-\frac{5}{2}$	0	1	$-\frac{1}{2}$	$\frac{5}{2}$
x_3	$\frac{1}{2}$	$\frac{3}{4}$	1	0	$\frac{1}{12}$	$\frac{1}{4}$
	7	$\frac{5}{2}$	0	0	$\frac{3}{2}$	$-\frac{5}{2}$

The auxiliary problem "Maximize $z = -y_1 - y_2 \ldots$" has $-\frac{5}{2} \neq 0$ as its optimal objective value.

The given problem has no feasible solution.

Supplementary problems

1. Maximize $z = -x_1 + 5x_2 + 3x_3$ subject to the constraints

$$2x_1 + x_2 + x_3 = 5$$
$$3x_1 + 2x_2 \geq 6$$
$$4x_1 + 3x_2 - x_3 \leq 7$$
$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$

phase 1, Tableau (1)

	x_1	x_2	x_3	x_4	x_5	y_1	y_2		θ-ratios
y_1	2	1	1	0	0	1	0	5	$\frac{5}{2}$
y_2	3	2	0	-1	0	0	1	6	$\frac{6}{3}$
x_5	(4)	3	-1	0	1	0	0	7	$\frac{7}{4}$
	-5	-3	-1	0	0	0	0	-11	

phase 1, Tableau (2)

	x_1	x_2	x_3	x_4	x_5	y_1	y_2		θ-ratios
y_1	0	$-\frac{1}{2}$	(3/2)	0	$-\frac{1}{2}$	1	0	$\frac{3}{2}$	1
y_2	0	$-\frac{1}{4}$	$\frac{3}{4}$	-1	$-\frac{3}{4}$	0	1	$\frac{3}{4}$	1
x_1	1	$\frac{3}{4}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	0	0	$\frac{7}{4}$	
	0	$\frac{3}{4}$	$-\frac{9}{4}$	1	$\frac{5}{4}$	0	0	$-\frac{9}{4}$	

Supplementary problem 1 (cont'd)

phase 1, Tableau ③ : we arbitrarily choose y_1 to exit.

	x_1	x_2	x_3	x_4	x_5	y_1	y_2	
x_3	0	$-\frac{1}{3}$	1	0	$-\frac{1}{3}$	$\frac{3}{3}$	0	1
y_2	0	0	0	−1	$-\frac{1}{2}$	$-\frac{1}{2}$	1	0
x_1	1	$\frac{2}{3}$	0	0	$\frac{1}{6}$	$\frac{1}{6}$	0	2
	0	0	0	1	$\frac{1}{2}$	$\frac{3}{2}$	0	0

phase 2, Tableau ① : we retain the basic artificial variable, y_2.

	x_1	x_2	x_3	x_4	x_5	y_2	
x_3	0	$-\frac{1}{3}$	1	0	$-\frac{1}{3}$	0	1
y_2	0	0	0	−1	$-\frac{1}{2}$	1	0
x_1	1	$\frac{2}{3}$	0	0	$\frac{1}{6}$	0	2
	0	$-\frac{20}{3}$	0	0	$-\frac{7}{6}$	0	1

phase 2, Tableau ② is optimal:

	x_1	x_2	x_3	x_4	x_5	y_2	
x_3	$\frac{1}{2}$	0	1	0	$-\frac{1}{4}$	0	2
y_2	0	0	0	−1	$-\frac{1}{2}$	1	0
x_2	$\frac{3}{2}$	1	0	0	$\frac{1}{4}$	0	3
	10	0	0	0	$\frac{1}{2}$	0	21

Supplementary problems

2. Maximize $z = -9x_1 - 4x_3$ subject to the constraints

$$9x_1 + x_2 + x_3 \geq 27$$
$$3x_1 + x_2 + 2x_3 = 9$$
$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$$

Supplementary problem 2 (cont'd)

phase 1, Tableau ①

	x_1	x_2	x_3	x_4	y_1	y_2		θ-ratios
y_1	9	1	1	-1	1	0	27	3
y_2	(3)	1	2	0	0	1	9	3
	-12	-2	-3	1	0	0	-36	

phase 1, Tableau ②: we arbitrarily choose y_2 to exit.

	x_1	x_2	x_3	x_4	y_1	y_2	
y_1	0	-2	-5	-1	1	-3	0
x_1	1	$\frac{1}{3}$	$\frac{2}{3}$	0	0	$\frac{1}{3}$	3
	0	2	5	1	0	4	0

phase 2, Tableau ①: we retain y_1.

	x_1	x_2	x_3	x_4	y_1	
y_1	0	(-2)	-5	-1	1	0
x_1	1	$\frac{1}{3}$	$\frac{2}{3}$	0	0	3
	0	-3	-2	0	0	-27

phase 2, Tableau ②: we exit y_1, since exiting x_1 would cause y_1 to take a positive value.

	x_1	x_2	x_3	x_4	y_1	
x_2	0	1	$\frac{5}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0
x_1	1	0	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{6}$	3
	0	0	$\frac{11}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$	-27

After deleting the y_1 column immediately after exit, this tableau is optimal.