pg.165 2.

Maximize $z' = 3w_1 + 8w_2$ subject to the constraints

$$\begin{aligned} w_1 + 2w_2 &\le 6 \\ 2w_1 + w_2 &\le 6 \\ w_1 + 4w_2 &\le 8 \\ w_1 + 9w_2 &\le 9, \quad w_1 \ge 0, \ w_2 \ge 0. \end{aligned}$$

pg.165 4.

Minimize $z' = 10w_1 - 5w_2 - 8w_3 + 15w_4 + 20w_5$ subject to the constraints

$$\begin{aligned} 4w_1 - 4w_2 - 3w_3 + 3w_4 + w_5 &\ge 2 \\ 2w_1 - 2w_2 - 5w_3 + 5w_4 + w_5 &\ge 1 \\ 5w_1 - 5w_2 - 4w_3 + 4w_4 + w_5 &\ge 3 \\ 5w_1 - 5w_2 - w_3 + w_4 + w_5 &\ge 4, \end{aligned}$$

$w_1 \ge 0$, $w_2 \ge 0$, $w_3 \ge 0$, $w_4 \ge 0$, w_5 unrestricted

or equivalently

Maximize $z' = -10w_1 + 5w_2 + 8w_3 - 15w_4 + 20w_5$ subject to the constraints

$$\begin{aligned} -4w_1 + 4w_2 + 3w_3 - 3w_4 + w_5 &\le -2 \\ -2w_1 + 2w_2 + 5w_3 - 5w_4 + w_5 &\le -1 \\ -5w_1 + 5w_2 + 4w_3 - 4w_4 + w_5 &\le -3 \\ -5w_1 + 5w_2 + w_3 - w_4 + w_5 &\le -4, \end{aligned}$$

$w_1 \ge 0$, $w_2 \ge 0$, $w_3 \ge 0$, $w_4 \ge 0$, w_5 unrestricted

pg.166 6.

Maximize $z' = 12w_1 - 6w_2$ subject to the constraints

$$\begin{aligned} 4w_1 - 3w_2 &\le 5 \\ 2w_1 - 2w_2 &\le 2 \\ w_1 - 3w_2 &= 6, \quad w_1 \ge 0, \ w_2 \ge 0. \end{aligned}$$

or equivalently

Minimize $z'' = -12w_1 + 6w_2$ subject to the constraints

$$\begin{aligned} -4w_1 + 3w_2 &\ge -5 \\ -2w_1 + 2w_2 &\ge -2 \\ -w_1 + 3w_2 &= -6, \quad w_1 \ge 0, \ w_2 \ge 0. \end{aligned}$$

pg. 183 9. The dual problem is:

Minimize $z' = 6w_1 + 12w_2 + 5w_3$ subject to the constraints

$$\begin{aligned} 2w_1 + 5w_2 \quad &\geq 9 \\ w_1 + 4w_2 + 2w_3 &\geq 14 \\ 3w_1 + w_2 \quad &\geq 7, \; w_1 \geq 0, w_2 \geq 0, w_3 \geq 0. \end{aligned}$$

If Kolman and Beck's assertion that $x_i \neq 0$ $(i = 1, 2, 3)$ is correct, then by complementary slackness, at dual optimality there is no slack in the first three constraints of the dual problem. The (unique) solution of $2w_1 + 5w_2 = 9$, $w_1 + 4w_2 + 2w_3 = 14$, $3w_1 + w_2 = 7$ is $w_1 = 2$, $w_2 = 1$, $w_3 = 4$, where $z' = 44$.

Direct substitution (in the primal problem) shows that $x_1 = \frac{5}{26}$, $x_2 = \frac{5}{2}$, $x_3 = \frac{27}{26}$ is feasible for the primal problem and that $z = 44$ there. The corollary of the weak duality theorem (ie, theorem 3.6) implies that the two solutions are optimal for their respective problems.

pg. 184 11. The dual problem is:

Minimize $z' = 12w_1 + 10w_2 + 10w_3$ subject to the constraints

$$\begin{aligned} 2w_1 + w_2 + 3w_3 &\geq 4 \\ 3w_1 + 4w_2 + w_3 &\geq 2 \\ w_1 + 2w_2 + w_3 &\geq 3, \\ w_1 \geq 0, w_2 \geq 0&, w_3 \geq 0. \end{aligned}$$

Direct substitution of $[x_1 \; x_2 \; x_3]^T = [2 \; 0 \; 4]^T$ into the primal constraints show there is slack in the first constraint (only). So at dual optimality, $w_1 = 0$.

pg 184 11. (cont'd.) Also by complementary slackness and since $x_1 \neq 0$, $x_3 \neq 0$, at dual optimality there is no slack in the first and third dual constraints. The solution of

$$2w_1 + w_2 + 3w_3 = 4$$
$$w_1 + 2w_2 + w_3 = 3$$

which satisfies $w_1 = 0$ is $w_1 = 0$, $w_2 = 1$, $w_3 = 1$, and this is optimal for the dual problem.
At their respective optimal solutions, both problems have the objective value 20.

supplementary problem a) The problem in tableau form is

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	
x_6	11	1	-1	-7	1	1	0	0	3
x_7	-15	-1	2	23	-3	0	1	0	2
x_8	-7	2	-2	-9	1	0	0	1	4
	76	-13	11	27	-2	0	0	0	0

b) Since the final basic variables are x_5, x_3, x_2, the matrix B is $\begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ 1 & -2 & 2 \end{bmatrix}$, from the x_5, x_3, x_2 columns of the above tableau. A routine matrix inversion shows that $B^{-1} = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & -2 \\ 4 & 1 & -1 \end{bmatrix}$.

The constraint part of the final tableau is then

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & -2 \\ 4 & 1 & -1 \end{bmatrix}\left[\begin{array}{cccccccc|c} 11 & 1 & -1 & -7 & 1 & 1 & 0 & 0 & 3 \\ -15 & -1 & 2 & 23 & -3 & 0 & 1 & 0 & 2 \\ -7 & 2 & -2 & -9 & 1 & 0 & 0 & 1 & 4 \end{array}\right] \text{ or}$$

Supplementary problem b) (cont'd)

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	
x_5	29	0	0	-5	1	2	0	-1	2
x_3	54	0	1	6	0	5	1	-2	9
x_2	36	1	0	4	0	4	1	-1	10

initial objective row

$$[\,76 \quad -13 \quad 11 \quad 27 \quad -2 \quad 0 \quad 0 \quad 0 \mid 0\,]$$

x_5-row

$$+2\times[\,29 \quad 0 \quad 0 \quad -5 \quad 1 \quad 2 \quad 0 \quad -1 \mid 2\,]$$

x_3-row

$$-11\times[\,54 \quad 0 \quad 1 \quad 6 \quad 0 \quad 5 \quad 1 \quad -2 \mid 9\,]$$

x_2-row

$$+13\times[\,36 \quad 1 \quad 0 \quad 4 \quad 0 \quad 4 \quad 1 \quad -1 \mid 10\,]$$

$$= [\,8 \quad 0 \quad 0 \quad 3 \quad 0 \quad 1 \quad 2 \quad 7 \mid 35\,],$$

the final objective row, in which the coefficients of x_5, x_3, and x_2 have now been eliminated.

The final tableau is:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	
x_5	29	0	0	-5	1	2	0	-1	2
x_3	54	0	1	6	0	5	1	-2	9
x_2	36	1	0	4	0	4	1	-1	10
	8	0	0	3	0	1	2	7	35

c) $w_1 = 1$, $w_2 = 2$, $w_3 = 7$