pg. 214 2. Tableau ① is given in the question. x_6 exits. By the ratio test x_5 enters and the pivot is "$-\frac{1}{3}$" to get tableau ②:

	x_1	x_2	x_3	x_4	x_5	x_6	
x_1	1	0	0	-1	0	3	3
x_2	0	1	0	1	0	-2	4
x_3	0	0	1	7	0	-24	10
x_5	0	0	0	0	1	-3	1
	0	0	0	1	0	3	39

The optimal (and feasible) solution is
$[x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T = [3 \ 4 \ 10 \ 0 \ 1 \ 0]$.

pg. 215 4. The x_6-row of the tableau represents the constraint $\frac{1}{3}x_4 + \frac{2}{3}x_5 + x_6 = -\frac{1}{3}$.

In view of the fact that the tableau represents a canonical problem (with $x_4 \geq 0$, $x_5 \geq 0$, $x_6 \geq 0$), the problem has no feasible solution.

pg. 215 8. Tableau ①

	x_1	x_2	x_3	x_4	x_5	x_6	
x_4	-1	2	-1	1	0	0	-4
x_5	-2	-1	1	0	1	0	-6
x_6	-3	0	-5	0	0	1	-15
	1	2	0	0	0	0	0

pg. 215 8. (cont'd) Tableau (2)

	x_1	x_2	x_3	x_4	x_5	x_6	
x_4	$-\frac{2}{5}$	2	0	1	0	$-\frac{1}{5}$	-1
x_5	$-\frac{13}{5}$	-1	0	0	1	$\frac{1}{5}$	-9
x_3	$\frac{3}{5}$	0	1	0	0	$-\frac{1}{5}$	3
	1	2	0	0	0	0	0

Tableau (3)

	x_1	x_2	x_3	x_4	x_5	x_6	
x_4	0	$\frac{28}{13}$	0	1	$-\frac{2}{13}$	$-\frac{3}{13}$	$\frac{5}{13}$
x_1	1	$\frac{5}{13}$	0	0	$-\frac{5}{13}$	$-\frac{1}{13}$	$\frac{45}{13}$
x_3	0	$-\frac{3}{13}$	1	0	$\frac{3}{13}$	$-\frac{2}{13}$	$\frac{12}{13}$
	0	$\frac{21}{13}$	0	0	$\frac{5}{13}$	$\frac{1}{13}$	$-\frac{45}{13}$

Optimal values: $x_1 = \frac{45}{13}$, $x_2 = 0$, $x_3 = \frac{12}{13}$.

pg. 223 2.

Eta vector $= \begin{bmatrix} -\frac{2}{3} \\ \frac{4}{3} \\ \frac{4}{9} \\ 0 \\ -\frac{5}{9} \end{bmatrix}$. Eta matrix $= \begin{bmatrix} 1 & -\frac{2}{3} & 0 & 0 & 0 \\ 0 & \frac{4}{3} & 0 & 0 & 0 \\ 0 & \frac{4}{9} & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -\frac{5}{9} & 0 & 0 & 1 \end{bmatrix}$.

pg. 223 4.

$$\text{Eta matrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 1 & 0 \\ 0 & -\frac{1}{6} & 0 & 1 \end{bmatrix}$$

(by the θ-ratio test, the pivot is 4 ; $\frac{1}{4} < \frac{2}{(2/3)}$)

New B^{-1}

$$= \text{Eta matrix} \cdot \text{current } B^{-1} = \begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ -1 & \frac{7}{2} & 1 & -\frac{5}{2} \\ 2 & -\frac{7}{6} & 2 & \frac{23}{6} \end{bmatrix}.$$

pg. 224 6. Initial data (not to be deleted):

$$A = \begin{array}{c} \begin{array}{ccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \end{array} \\ \begin{bmatrix} 2 & 1 & 1 & 2 & 1 & 0 & 0 \\ 3 & 5 & 2 & 3 & 0 & 1 & 0 \\ 3 & 2 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}, \quad b = \begin{bmatrix} 18 \\ 24 \\ 12 \end{bmatrix}$$

$$c^T = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Tableau 1. Current basic variables $= \{x_5, x_6, x_7\}$ (the original slack variables).

pg. 224 6. (cont'd)

Current $B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Current $c_B^T = [0\ 0\ 0]$.

Current $w^T = c_B^T B^{-1} = [0\ 0\ 0]$.

Current objective row $= w^T A - c$

$$= \begin{array}{ccccccc} [-1 & -2 & -3 & -1 & 0 & 0 & 0]. \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \end{array}$$

x_3 enters.

Current pivotal column $= B^{-1}A_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$. $B^{-1}b = \begin{bmatrix} 18 \\ 24 \\ 12 \end{bmatrix}$

θ-ratios are $\frac{18}{1}, \frac{24}{2}, \frac{12}{1}$. Either x_6 or x_7 may exit. We choose x_7.

Tableau 2. New basic variables are $\{x_5, x_6, x_3\}$.

New B^{-1} = eta matrix $\cdot$ current $B^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$.

New $c_B^T = [0\ 0\ 3]$, new $w^T = c_B^T B^{-1} = [0\ 0\ 3]$,

and new objective row $= w^T A - c^T$

$$= [8\ 4\ 0\ 2\ 0\ 0\ 3].$$

Tableau 2 is optimal, $(B^{-1}b)^T = [6\ 0\ 12]$, and this gives the optimal solution

$x_1 = 0, x_2 = 0, x_3 = 12, x_4 = 0, x_5 = 6, x_6 = 0, x_7 = 0$,

with optimal objective value $w^T b = 36$.

pg. 224 8. Initial data:

$$A = \begin{array}{c} \begin{array}{ccccccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} \end{array} \\ \begin{bmatrix} 3 & 1 & 1 & 1 & 2 & 3 & 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 2 & 0 & 5 & 1 & 2 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 & 3 & 0 & 1 & 3 & 0 & 0 & 1 \end{bmatrix} \end{array}, \quad b = \begin{bmatrix} 12 \\ 15 \\ 18 \end{bmatrix}$$

$$c^T = [3 \quad 2 \quad 0 \quad 0 \quad 4 \quad 1 \quad 0 \quad 2 \quad 0 \quad 0 \quad 0]$$

Tableau 1. Current basic variables $= \{x_9, x_{10}, x_{11}\}$.

Current $B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Current $c_B^T = [0 \quad 0 \quad 0]$.

Current $w^T = c_B^T B^{-1} = [0 \quad 0 \quad 0]$.

Current objective row $= w^T A - c$

$$= \begin{array}{c} [-3 \quad -2 \quad 0 \quad 0 \quad -4 \quad -1 \quad 0 \quad -2 \quad 0 \quad 0 \quad 0] \\ \begin{array}{ccccccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} \end{array} \end{array}$$

x_5 enters.

Current pivotal column $= B^{-1}A_5 = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$. $B^{-1}b = \begin{bmatrix} 12 \\ 15 \\ 18 \end{bmatrix}$

θ-ratios are $\frac{12}{2}$, $\cancel{\frac{15}{0}}$, $\frac{18}{3}$. Either x_9 are x_{11} may exit. We choose x_9.

pg. 224 8. (cont'd)

Tableau 2. New basic variables are $\{x_5, x_{10}, x_{11}\}$.

New B^{-1} = eta matrix · current B^{-1} = $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{3}{2} & 0 & 1 \end{bmatrix}$.

New $C_B^T = [4 \ \ 0 \ \ 0]$, new $w^T = C_B^T B^{-1} = [2 \ \ 0 \ \ 0]$,

and new objective row $= w^T A - C^T$
$= [3 \ \ 0 \ \ 2 \ \ 2 \ \ 0 \ \ 5 \ \ 0 \ \ 0 \ \ 2 \ \ 0 \ \ 0]$

Tableau (2) is optimal, $B^{-1}b = \begin{bmatrix} 6 \\ 15 \\ 0 \end{bmatrix} \begin{array}{l} \leftarrow x_5 \\ \leftarrow x_{10} \\ \leftarrow x_{11} \end{array}$

and the optimal objective value is $w^T b = 24$.