APM 236H1F term test 1

15 October, 2008

FAMILY NAME ________________________

GIVEN NAME(S) ________________________

STUDENT NUMBER ________________________

SIGNATURE ________________________

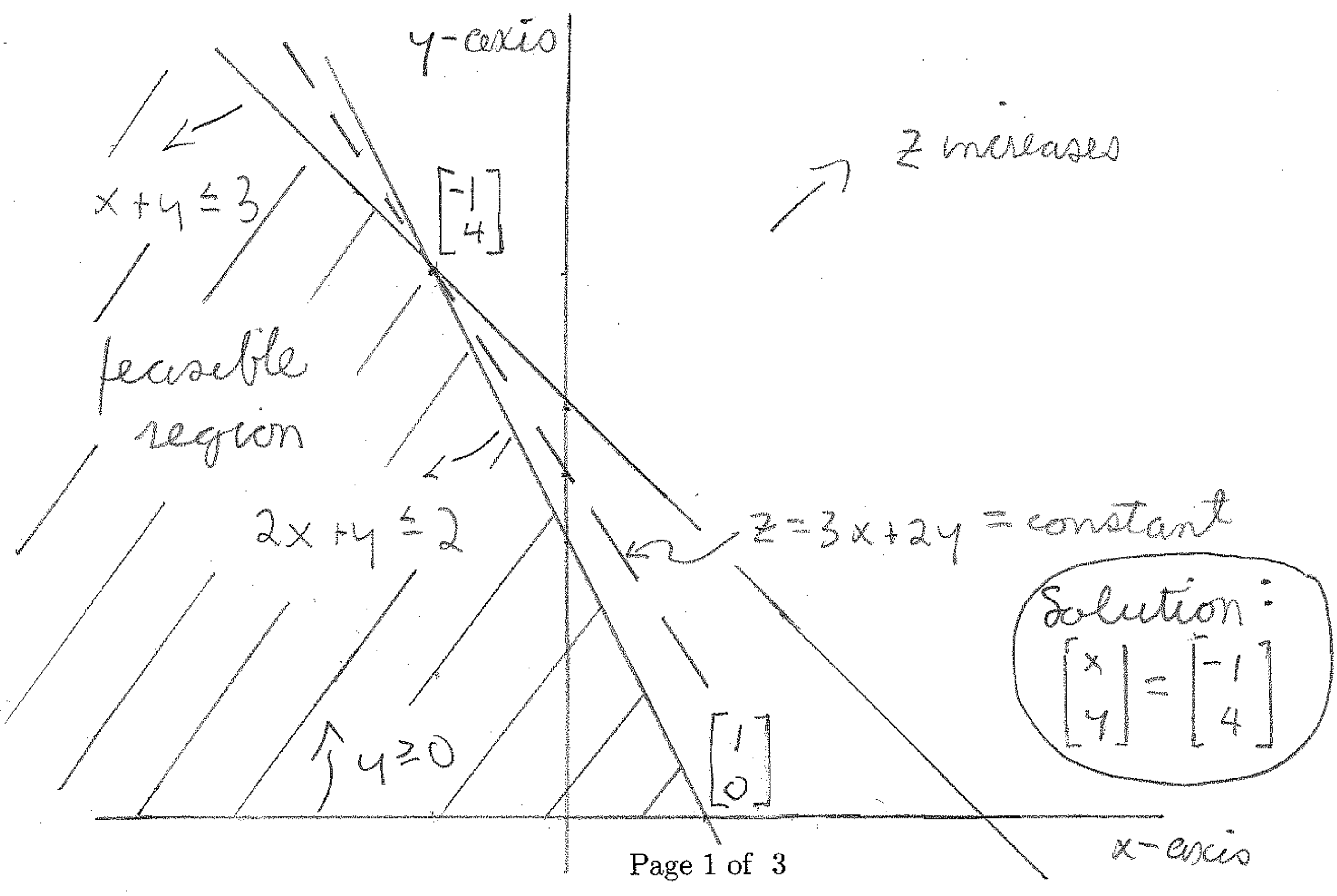
Instructions: No calculators or other aids allowed.

This test has 3 questions whose values are given immediately after the question numbers. Total marks = 40.

Write solutions in the spaces provided, using the backs of the pages if necessary. (Suggestion: If you have to continue a question, you may use the back of the **previous** page.) Aspects of any question which are indicated in **boldface** will be regarded as crucial during grading. **Show your work.**

The duration of this test is 50 minutes.

1. (13 marks) **Solve** the following problem **graphically**: Maximize $z = 3x + 2y$ subject to the constraints $\begin{array}{rcrcr} x & + & y & \leq & 3 \\ 2x & + & y & \leq & 2 \end{array}$, x unrestricted, $y \geq 0$.



2. (13 marks). Mr. Fillmore owns three furniture factories, which make tables and chairs only, on Harrison Avenue, Tyler Road, and Polk Street. It costs \$7 to produce a table in the Harrison Avenue factory, while it costs \$5 to produce a chair there. Similar production costs in the other factories are: \$10 per table and \$6 per chair in the Tyler Road factory and \$11 per table and \$4 per chair in the Polk Street factory.

The factories have separate labour unions and the contract at the Harrison Avenue factory requires that the cost of producing chairs there must be at most 70% of the total cost of producing all furniture (tables and chairs) at the Harrison Avenue factory. Also, the number of tables produced in the Polk Street factory must be at least 40% of the total number of tables produced in all three factories.

Mr.Fillmore has received an order which he must fill in six weeks, for 200 tables (exactly) and 800 chairs (exactly). In that time, the Tyler Road factory can make at most 300 items of furniture (tables and chairs total) but the other factories have no similar restrictions.

Set up a linear programming problem in **general form** (that is, in which no decision variable appears on the right hand side of any constraint), whose solution will tell Mr. Fillmore how many tables and chairs he should make in each factory to fill the order while minimizing his total production cost. You may assume the factories can make fractional parts of tables and chairs (that is, the outputs of the factories need not be integers). After setting up the linear programming problem, do not solve it.

Let x_{ij} ($i=1,2$; $j=1,2,3$) denote the number of tables and chairs made in each factory, as indicated by the chart:

	Harrison	Tyler	Polk
tables	x_{11}	x_{12}	x_{13}
chairs	x_{21}	x_{22}	x_{23}

(The conditions involving percentages are: $5x_{21} \le .7(7x_{11}+5x_{21})$ and $x_{13} \ge .4(x_{11}+x_{12}+x_{13})$.) A linear programming model is:

Minimize $z = 7x_{11} + 10x_{12} + 11x_{13} + 5x_{21} + 6x_{22} + 4x_{23}$ s.t.

$$
\begin{array}{lcl}
4.9x_{11} - 1.5x_{21} & \ge & 0 \\
.4x_{11} + .4x_{12} - .6x_{13} & \le & 0 \\
x_{11} + x_{12} + x_{13} & = & 200 \\
x_{21} + x_{22} + x_{23} & = & 800 \\
x_{12} + x_{22} & \le & 300
\end{array}
$$

$x_{ij} \ge 0$ for $i=1,2$; $j=1,2,3$.

3. (14 marks) Consider the following linear programming problem (in standard form):

Maximize $z = x_1 + 2x_2 - 3x_3$ subject to the constraints

$$\begin{array}{rcrcrcr} & & x_2 & & & \leq & 4 \\ 2x_1 & + & 3x_2 & - & 4x_3 & \leq & 24 \end{array}, \quad x_1 \geq 0, x_2 \geq 0, x_3 \geq 0\,.$$

(a) (2 marks) Put the problem in **canonical form**.

(b) (7 marks) Find **all basic solutions** (feasible and infeasible) of the **canonical form** of the problem.

(c) (3 marks) Find **all extreme points** of the feasible region of the **standard form** of the problem.

(d) (2 marks) **Solve** the **standard form** of the problem. You may assume the problem has an optimal solution.

(a) Maximize $Z = x_1 + 2x_2 - 3x_3$ s.t.

$$\begin{array}{l} x_2 + x_4 = 4 \\ 2x_1 + 3x_2 - 4x_3 + x_5 = 24 \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0. \end{array}$$

(x_4 and x_5 are slacks.)

(b) The coefficient matrix of the equality constraints is

$$\begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array}$$
$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 2 & 3 & -4 & 0 & 1 \end{bmatrix}.$$

Since any 2 of $\{A_1, A_3, A_5\}$ is linearly dependent a basic solution can have at most 1 of $\{x_1, x_3, x_5\}$ as a basic variable. Basic solutions are

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 12 \\ 0 \\ 0 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 8 \\ 0 \\ -4 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 0 \\ 0 \\ 12 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -6 \\ 4 \\ 0 \end{bmatrix}, \text{and} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \\ 24 \end{bmatrix}.$$

basic variables $\rightarrow$ $\{x_1, x_2\}$ $\{x_1, x_4\}$ $\{x_2, x_3\}$ $\{x_2, x_4\}$ $\{x_2, x_5\}$ $\{x_3, x_4\}$ $\{x_4, x_5\}$

(c) Discarding the infeasible solutions and dropping slacks, extreme points are

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}, \text{and} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

(d) $Z = 14, 12, 8$, and 0 are the respective objective values. This maximization problem has solution $\begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}$.