APM 236H1F term test 2

5 November, 2012

FAMILY NAME ______________________________

GIVEN NAME(S) ______________________________

STUDENT NUMBER ______________________________

SIGNATURE ______________________________

Instructions: No calculators or other aids allowed.

This test has 3 questions whose values are given immediately after the question numbers. Total marks = 40.

Write solutions in the spaces provided, using the backs of the pages if necessary. (Suggestion: If you have to continue a question, you may use the back of the **previous** page.) Aspects of any question which are indicated in **boldface** will be regarded as crucial during grading. **Show your work.**

The duration of this test is 50 minutes.

1. (13 marks) **Solve** the problem: Maximize $z = -x_1 + 4x_2 + x_3$ subject to the constraints

$$\begin{array}{rcrcrcl} 3x_1 & + & 3x_2 & + & x_3 & \leq & 9 \\ -x_1 & + & 2x_2 & - & x_3 & \leq & 7 \\ -2x_1 & - & x_2 & + & x_3 & \leq & 1 \end{array} \quad , \quad x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

x_4, x_5, and x_6 are slack variables.

Tableau 1

	x_1	x_2	x_3	x_4	x_5	x_6	
x_4	3	3	1	1	0	0	9
x_5	-1	2	-1	0	1	0	7
x_6	-2	-1	1	0	0	1	1
	1	-4	-1	0	0	0	0

Tableau 2

	x_1	x_2	x_3	x_4	x_5	x_6	
x_2	1	1	$\frac{1}{3}$	$\frac{1}{3}$	0	0	3
x_5	-3	0	$-\frac{5}{3}$	$-\frac{2}{3}$	1	0	1
x_6	-1	0	$\frac{4}{3}$	$\frac{1}{3}$	0	1	4
	5	0	$\frac{1}{3}$	$\frac{4}{3}$	0	0	12

2. (13 marks) Suppose in solving a certain canonical linear programming problem by the simplex method we encounter the following tableau:

	x_1	x_2	x_3	x_4	
x_4	0	3	-2	1	8
x_1	1	2	-7	0	4
	0	-6	-5	0	0

Now let M be any fixed non-negative number. Find $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ (depending on M), which is **feasible** for the problem, such that, at $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$, the problem has **objective value greater than or equal to** M.

The tableau represents the problem

Maximize $z = 6x_2 + 5x_3$ s.t.

$$3x_2 - 2x_3 + x_4 = 8$$
$$x_1 + 2x_2 - 7x_3 = 4$$
$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0.$$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7M+4 \\ 0 \\ M \\ 2M+8 \end{bmatrix}$ is feasible ($M \geq 0$) and $z = 5M \geq M$ at this point.

(In fact, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \frac{7}{5}M+4 \\ 0 \\ \frac{1}{5}M \\ \frac{2}{5}M+8 \end{bmatrix}$ suffices; $z = M$ here.)

3. (14 marks) **Solve** the problem: Maximize $z = -x_1 - 4x_2 + x_3$ subject to the constraints

$$\begin{array}{rcrcrcc} x_1 & + & 2x_2 & - & 2x_3 & = & 2 \\ x_1 & + & 2x_2 & + & x_3 & \geq & 5 \end{array}, \quad x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

x_4 is slack and y_1, y_2 are artificial.

phase 1, Tableau 1

	x_1	x_2	x_3	x_4	y_1	y_2	
y_1	1	2	-2	0	1	0	2
y_2	1	2	1	-1	0	1	5
	-2	-4	1	1	0	0	-7

phase 1, Tableau 2

	x_1	x_2	x_3	x_4	y_1	y_2	
x_2	$\frac{1}{2}$	1	-1	0	$\frac{1}{2}$	0	1
y_2	0	0	3	-1	-1	1	3
	0	0	-3	1	2	0	-3

phase 1, Tableau 3

	x_1	x_2	x_3	x_4	y_1	y_2	
x_2	$\frac{1}{2}$	1	0	$-\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{3}$	2
x_3	0	0	1	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	1
	0	0	0	0	1	1	0

phase 2, tableau 1

	x_1	x_2	x_3	x_4	
x_2	$\frac{1}{2}$	1	0	$-\frac{1}{3}$	2
x_3	0	0	1	$-\frac{1}{3}$	1
	-1	0	0	1	-7

	x_1	x_2	x_3	x_4	
x_1	1	2	0	$-\frac{2}{3}$	4
x_3	0	0	1	$-\frac{1}{3}$	1
	0	2	0	$\frac{1}{3}$	-3