APM 236H1F term test 2

5 November, 2014

FAMILY NAME ____________________________________

GIVEN NAME(S) ____________________________________

STUDENT NUMBER ____________________________________

SIGNATURE ____________________________________

Instructions: No calculators or other aids allowed.

This test has 3 questions whose values are given immediately after the question numbers. Total marks = 40.

Write solutions in the spaces provided, using the backs of the pages if necessary. (Suggestion: If you have to continue a question, you may use the back of the **previous** page.) Aspects of any question which are indicated in **boldface** will be regarded as crucial during grading. **Show your work.**

The duration of this test is 50 minutes.

1. (13 marks) For the following problem, find a **feasible solution** where the **objective value is greater than 1000**. Maximize $z = 2x_1 + x_2$ subject to the constraints

$$\begin{array}{rcrcrcr} 4x_1 & + & 2x_2 & - & 5x_3 & \leq & 10 \\ x_1 & - & x_2 & + & 2x_3 & \leq & 1 \end{array}, \quad x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

Tableau 1

	x_1	x_2	x_3	x_4	x_5	
x_4	4	2	-5	1	0	10
x_5	1	-1	2	0	1	1
	-2	-1	0	0	0	0

Tableau 2

	x_1	x_2	x_3	x_4	x_5	
x_4	0	6	-13	1	-4	6
x_1	1	-1	2	0	1	1
	0	-3	4	0	2	2

Tableau 3

	x_1	x_2	x_3	x_4	x_5	
x_2	0	1	$-\frac{13}{6}$	$\frac{1}{6}$	$-\frac{2}{3}$	1
x_1	1	0	$-\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	2
	0	0	$-\frac{5}{2}$	$\frac{1}{2}$	0	5

If $M > 398$, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2+\frac{1}{6}M \\ 1+\frac{13}{6}M \\ M \end{bmatrix}$ is feasible with $Z = 5+\frac{5}{2}M > 1000$.

(Just one example (using $M=600$) of a correct answer is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 102 \\ 1301 \\ 600 \end{bmatrix}$.)

2. (13 marks) Suppose in solving a linear programming problem by the simplex method we encounter a tableau, part of which is given below, where $a_1 > 0$ and $a_m > 0$.

	x_n	
x_1	a_1	b_1
$\vdots$	$\vdots$	$\vdots$
x_m	a_m	b_m
	-1	0

In the next iteration of the simplex method, x_n will enter. Now suppose that the θ ratio for the x_1 row is less than the θ ratio for the x_m row but, contrary to the rules of the simplex method, we exit x_m. **Prove** that the next tableau will be infeasible.

Entering x_n and exiting x_m will produce a tableau where x_1 has the value $b_1 - a_1 \frac{b_m}{a_m} = a_1\left(\frac{b_1}{a_1} - \frac{b_m}{a_m}\right)$.

Since the θ-ratios $\frac{b_1}{a_1}$ and $\frac{b_m}{a_m}$ satisfy

$\frac{b_1}{a_1} < \frac{b_m}{a_m}$, we have $\frac{b_1}{a_1} - \frac{b_m}{a_m} < 0$

(while $a_1 > 0$), so that the value of x_1 will be <u>negative</u>.

3. (14 marks) **Solve** the problem: Maximize $z = x_2 - 2x_3$ subject to the constraints

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 4x_3 & = & -4 \\ 2x_1 & + & x_2 & + & 3x_3 & \geq & 7 \end{array}, \; x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

(x_4 is slack; y_1 and y_2 are artificial.)

phase 1, Tableau 1

	x_1	x_2	x_3	x_4	y_1	y_2	
y_1	-1	2	-4	0	1	0	4
y_2	2	1	3	-1	0	1	7
	-1	-3	1	1	0	0	-11

phase 1, Tableau 2

	x_1	x_2	x_3	x_4	y_1	y_2	
x_2	$-\frac{1}{2}$	1	-2	0	$\frac{1}{2}$	0	2
y_2	$\frac{5}{2}$	0	5	-1	$-\frac{1}{2}$	1	5
	$-\frac{5}{2}$	0	-5	1	$\frac{3}{2}$	0	-5

phase 1, Tableau 3

	x_1	x_2	x_3	x_4	y_1	y_2	
x_2	$\frac{1}{2}$	1	0	$-\frac{2}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	4
x_3	$\frac{1}{2}$	0	1	$-\frac{1}{5}$	$-\frac{1}{10}$	$\frac{1}{5}$	1
	0	0	0	0	1	1	0

phase 2, Tableau 1

	x_1	x_2	x_3	x_4	
x_2	$\frac{1}{2}$	1	0	$-\frac{2}{5}$	4
x_3	$\frac{1}{2}$	0	1	$-\frac{1}{5}$	1
	$-\frac{1}{2}$	0	0	0	2

phase 2, Tableau 2

	x_1	x_2	x_3	x_4	
x_2	0	1	-1	$-\frac{1}{5}$	3
x_1	1	0	2	$-\frac{2}{5}$	2
	0	0	1	$-\frac{1}{5}$	3

The x_4-column indicates the problem is unbounded (that is, unbounded above: max problem)