APM 236H1F term test 2

11 November, 2009

FAMILY NAME ____________________

GIVEN NAME(S) ____________________

STUDENT NUMBER ____________________

SIGNATURE ____________________

Instructions: No calculators or other aids allowed.

This test has 3 questions whose values are given immediately after the question numbers. Total marks = 40.

Write solutions in the spaces provided, using the backs of the pages if necessary. (Suggestion: If you have to continue a question, you may use the back of the **previous** page.) Aspects of any question which are indicated in **boldface** will be regarded as crucial during grading. **Show your work.**

The duration of this test is 50 minutes.

1. (13 marks) **Solve** the problem: Maximize $z = -x_1+3x_2+2x_3$ subject to the constraints

$$\begin{array}{rcrcrcl} 6x_1 & - & 3x_2 & - & 2x_3 & \leq & 6 \\ -x_1 & + & x_2 & + & x_3 & \leq & 1 \\ 2x_1 & + & 2x_2 & + & 3x_3 & \leq & 6 \end{array} \quad , \quad x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

Tableau 1

slacks

	x_1	x_2	x_3	x_4	x_5	x_6	
x_4	6	-3	-2	1	0	0	6
x_5	-1	1	1	0	1	0	1
x_6	2	2	3	0	0	1	6
	1	-3	-2	0	0	0	0

Tableau 2

	x_1	x_2	x_3	x_4	x_5	x_6	
x_4	3	0	1	1	3	0	9
x_2	-1	1	1	0	1	0	1
x_6	4	0	1	0	-2	1	4
	-2	0	1	0	3	0	3

Tableau 3

	x_1	x_2	x_3	x_4	x_5	x_6	
x_6	0	0	$\frac{1}{4}$	1	$\frac{3}{2}$	$-\frac{3}{4}$	6
x_2	0	1	$\frac{5}{4}$	0	$\frac{1}{2}$	$\frac{1}{4}$	2
x_1	1	0	$\frac{1}{4}$	0	$-\frac{1}{2}$	$\frac{1}{4}$	1
	0	0	$\frac{3}{2}$	0	2	$\frac{1}{2}$	5

2. (14 marks) **Solve** the problem: Maximize $3x_1 + x_2 - 2x_3$ subject to the constraints

$$\begin{array}{rcrcrcr} x_1 & + & x_2 & - & 2x_3 & \leq & -2 \\ 3x_1 & + & 4x_2 & - & 4x_3 & = & 0 \end{array} \quad , \quad x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

phase 1, Tableau 1

	x_1	x_2	x_3	x_4	y_1	y_2	
y_1	-1	-1	2	-1	1	0	2
y_2	3	4	-4	0	0	1	0
	-2	-3	2	1	0	0	-2

phase 1, Tableau 2

	x_1	x_2	x_3	x_4	y_1	y_2	
y_1	$-\frac{1}{4}$	0	1	-1	1	$\frac{1}{4}$	2
x_2	$\frac{3}{4}$	1	-1	0	0	$\frac{1}{4}$	0
	$\frac{1}{4}$	0	-1	1	0	$\frac{3}{4}$	-2

phase 1, Tableau 3

	x_1	x_2	x_3	x_4	y_1	y_2	
x_3	$-\frac{1}{4}$	0	1	-1	1	$\frac{1}{4}$	2
x_2	$\frac{1}{2}$	1	0	-1	1	$\frac{1}{2}$	2
	0	0	0	0	1	1	0

(In phase 1, x_4 is the slack in $-x_1 - x_2 + 2x_3 \geq 2$; y_1 and y_2 are artificial)

phase 2, Tableau 1

	x_1	x_2	x_3	x_4	
x_3	$-\frac{1}{4}$	0	1	-1	2
x_2	$\frac{1}{2}$	1	0	-1	2
	-2	0	0	1	-2

phase 2, Tableau 2

	x_1	x_2	x_3	x_4	
x_3	0	$\frac{1}{2}$	1	$-\frac{3}{2}$	3
x_1	1	2	0	-2	4
	0	4	0	-3	6

The x_4- column indicates this problem is unbounded above.

3. (13 marks) Suppose in solving a certain canonical linear programming problem by the simplex method we encounter the following tableau:

	x_1	x_2	x_3	x_4	
x_4	0	3	−2	1	8
x_1	1	2	−7	0	4
	0	−6	−5	0	0

Now let M be any fixed, but unspecified, non-negative number. Find $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ (depending on M), which is **feasible** for the problem, such that, at $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$, the problem has **objective value greater than or equal to** M.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7M+4 \\ 0 \\ M \\ 2M+8 \end{bmatrix}$$

is feasible with $z = 6x_2 + 5x_3$

$= 5M$

$\geq M$