CONTROL THEORY – MAT1840HF/MAT482H1F

Fall 2021

Time/location: Tu 3-5pm and Th 4-5pm
in BF 323 (Bancroft Building).

Instructor: Boris Khesin

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Office hours: TBA

Course description:

The course focuses on the key notions of Calculus of Variations and Optimal Control Theory: key examples of variational problems, (un)constrained optimization, first- and second-order conditions, Euler-Lagrange equation, variational problems with constraints, examples of control systems, the maximum principle, the Hamilton-Jacobi-Bellman equation (time permitting), holonomic and nonholonomic constraints, Frobenius theorem, Riemannian and sub-Riemannian geodesics.

Textbooks:

1) D. Liberzon "Calculus of Variations and Optimal Control Theory: A Concise Introduction" 2012, Princeton Univ. Press (chapters 1,2,3 and partially chapters 4,5,7)

2) A. Agrachev, D. Barilari, and U. Boscain "'A Comprehensive Introduction to Sub-Riemannian Geometry" 2019, Cambridge Univ. Press (chapter 2 and partially chapters 1,3)

Preliminary versions of the books can be downloaded from:
`http://liberzon.csl.illinois.edu/teaching/cvoc.pdf`
`https://www.imj-prg.fr/~davide.barilari/2016-11-21-ABB.pdf`

Course Website:

The website for the course is `http://www.math.toronto.edu/khesin/teaching/control/controltheory21syl.html`

Homework Assignments:

There will be 3-4 assignments and a final individual project, which together constitute the full course mark. No late assignments will be accepted.

Note: You must write your solutions yourself, in your own words. If your solution is aided by information from textbooks or online sources, you must properly quote these references.

Code of Behaviour / Plagiarism:

Students should become familiar with and are expected to adhere to the Code of Behaviour on Academic Matters which can be found at:
`http://www.governingcouncil.utoronto.ca/policies/behaveac.htm`

Course Outline:

The following is a tentative outline of the material to be covered.

Sep 9-16: Introduction: examples, (un)constrained optimization, Lagrange multipliers first and second variations.

Sep 21 - Oct 7: Calculus of variations: examples (Dido problem, catenary, brachistochrone), weak and strong extrema, Euler-Lagrange equation, introduction to Hamiltonian formalism, integral and non-integral constraints.

Oct 12-19: From calculus of variations to optimal control: control system, cost functional, target set.

Oct 21 - Nov 4: The maximum principle: statement, ideas of proof, weak form, examples.

Nov 16 - Dec 8: Lie brackets of vector fields, Frobenius theorem, nonholonomic constraints, examples (ball rolling, car parking), Chow-Rashevskii theorem, sub-Riemannian metrics.

Prerequisites:

MAT357H (recommended)/ MAT337H,
MAT351Y (recommended)/APM346H,
MAT267H (recommended)/ MAT244H.