These homework problems are meant to expand your understanding of what goes on during class. Any you turn in will be graded and returned to you. Answers may or may not be posted on the web, depending on demand.

1. Prove that Euclid's Postulate 5 and Playfair's Postulate are equivalent. That is, assuming "absolute geometry" (Euclid's first four postulates), prove that...
   (a) Postulate 5 implies Playfair's Postulate, and
   (b) Playfair's Postulate implies Euclid's Postulate 5.
2. Prove that the sum of angles in a quadrilateral is at most 360°. (Do not assume Euclid's Postulate 5 – assume only "absolute geometry.")

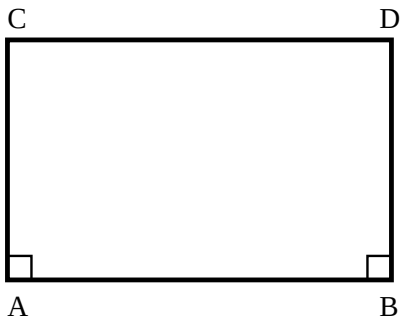

Figure 1: A quadrilateral

3. For this problem, consider the above quadrilateral in the context of absolute geometry.
   (a) If $AC = BD$, this is called a *Saccheri quadrilateral*. Prove in this case that $\angle C = \angle D$.
   (b) If, on the other hand, $AC < BD$, show that $\angle C > \angle D$.
   (c) Suppose now that $\angle C > \angle D$. Show that $AC < BD$. (Hint: work by contradiction. That is, assume that $AC \geq BD$ and show this implies that $\angle C \leq \angle D$. Perhaps it's easier to do two cases: $AC = BD$ and $AC > BD$.)

   Note that in parts (b) and (c) you've proved a statement and it's converse. That is, you've shown that $AC < BD$ if and only if $\angle C > \angle D$.)
4. For this problem, you may use only results from absolute geometry . Consider a Lambert quadrilateral, which is the quadrilateral above with $\angle C$ also a right angle (so three of the four angles are assumed to be right angles).
   (a) Show that the fourth angle, $\angle D$, is never greater than 90°.
   (b) Show that if $\angle D$ is also a right angle, then opposite sides of Lambert's quadrilateral are congruent.
   (c) Show that if $\angle D$ is acute (less than 90°), then $CD > AB$ and $BD > AC$.

5. Using the previous problem and the fact that the angle of parallelism is acute (under 90°), show that, in hyperbolic geometry, the distance between parallel lines is decreasing.

6. What can you say about the sum of the angles of a quadrilateral in...

   (a) in absolute geometry?

   (b) in hyperbolic geometry?