These homework problems are meant to expand your understanding of what goes on during class. Any you turn in will be graded and returned to you. Answers may or may not be posted on the web, depending on demand.

1. When discussing tilings of the hyperbolic plane by regular $n$-gons, we said that we would need at least 4 $n$-gons at each vertex when $n = 5$ or 6, and at least 3 $n$-gons when $n \geq 7$. Explain why this is so.

2. In spherical geometry, the angle excess plays the same role as the angle defect does in hyperbolic geometry. Let $E(\triangle) = S(\triangle) - 180°$ be the angle excess, where $S(\triangle)$ is (as usual) the angle sum in degrees.

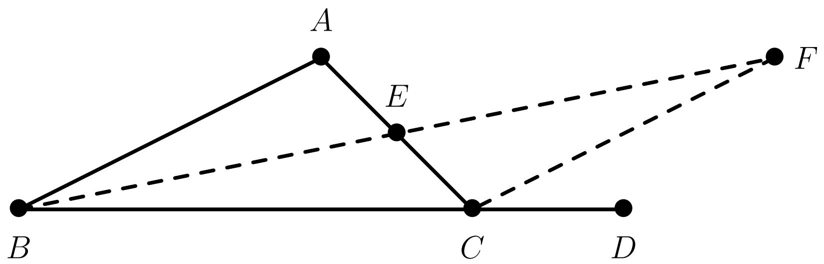
   (a) Explain why the angle excess is additive. That is, if a triangle $\triangle_1$ is formed by joining two other triangles $\triangle_2$ and $\triangle_3$ along an edge, show that $E(\triangle_1) = E(\triangle_2) + E(\triangle_3)$.

   (b) Assume that the area of a triangle $A(\triangle)$ is given by a constant multiple of the excess $E(\triangle)$; that is, assume that $A(\triangle) = C \cdot E(\triangle)$. Find the constant $C$. Your answer should involve the radius $r$ of the sphere.
   **Hint:** Consider the $90° - 90° - 90°$ triangle that is one-eighth of a sphere. What is its area?

3. Here is Euclid's proof of Proposition 16 (Book I):

   > In any triangle, if one of the sides is produced, then the exterior angle is greater than either of the interior and opposite angles.

   We draw a picture to make clear what Euclid is saying:




   The claim is that both interior angles $\angle BAC$ and $\angle ABC$ are smaller than the exterior angle $\angle ACD$.

   (a) Euclid's proof begins by drawing the bisector $BE$ of $AC$, and extending it to $F$ so that $BE = EF$. Use the picture to finish this proof that $\angle BAC < \angle ACD$.

   (b) Explain why $\angle ABC < \angle ACD$ as well.

   (c) This argument fails when it is made on the sphere. Explain why.
   **Hint:** What happens when $BF$ is a semi-circle. When $BF$ is *more* than a semi-circle?