Rules Of Logarithms

Rules of Exponents.

$\frac{1}{a^k} = a^{-k}$ $\quad a^k\, a^n = a^{k+n}$ $\quad \frac{a^k}{a^n} = a^{k-n}$ $\quad \left(\frac{a}{b}\right)^k = \frac{a^k}{b^k}$ $\quad (a^k)^n = a^{kn}$ $\quad \sqrt[k]{a} = a^{1/k}$

Rewrite each of the following expressions in the form $a^{\square}\, b^{\square}\, c^{\square}$.

1 $\quad \frac{a^7\, b^2}{a\, b\, c}$

2 $\quad \left(\frac{a^t\, b^5}{c^r}\right)\left(\frac{a^2\, c^3}{b^2}\right)$

3 $\quad \frac{a^2\, b^{-2}\, \sqrt{c}}{a^{3/2}\, b^{-3}\, c^5}$

4 $\quad \left(\frac{a^3\sqrt{b}}{c^7}\right)^5$

Exponential and Logarithmic Functions.

A *logarithm* is the inverse of an exponential. That is, $\log_a a^x = x$ for any positive $a \neq 1$, and $a^{\log_a x} = x$. We usually use a *base* of e, which is natural constant (that is, a number with a letter name, just like π). The number e is approximately 2.7182818284590452354. The logarithm we usually use is log base e, written $\log_e(x)$ or (more often) $\ln(x)$, and called the *natural logarithm* of x.

Rules of Logarithms.

- **Definition:** $c = \log_b(a) \iff a = b^c$
- **The Big One:** $\ln(x^y) = y \cdot \ln(x)$ or $\log_a(x^y) = y \cdot \log_a(x)$
- **Others:** $\log_a(r \cdot s) = \log_a(r) + \log_a(s)$ $\qquad \log_a(r/s) = \log_a(r) - \log_a(s)$

  $\log_a(b) = \frac{\log_x(b)}{\log_x(a)}$, for any x

Solve for t (algebraically, not numerically) in the following equations.

5 $\quad 200 = 5\, t^3$

6 $\quad 800 = 4 \cdot 7^t$

7 $\quad 400 = 200 + 3 \cdot 2^t$

8 $\quad 432 = 100e^{0.6t}$

9 $\quad \log_2(t) = 6$

10 $\quad \ln(t^2) = 30$

Functions of Exponential Type.

A function is said to be of exponential type if it can be written in the form

$y = a \cdot b^t$ $\qquad$ where a and b are constants.

If we are given two data points, (two pairs of t and y values), we can determine the constants a and b by solving a system of two equations.

Example: Given that $200 = a \cdot b^2$ and $450 = a \cdot b^7$, we divide the second equation by the first to get: $\frac{450}{200} = \frac{b^7}{b^2}$ and so $9/4 = b^5$, giving $b = \sqrt[5]{9/4}$.

Substituting that into the first equation gives $a = \frac{200}{(9/4)^{2/5}} = 200 \cdot (9/4)^{-2/5}$.

Find a and b given that:

11 $\quad 30 = a \cdot b^5$ and $80 = a \cdot b^9$

12 $\quad 1.5 = a \cdot b^{24}$ and $2.3 = a \cdot b^{36}$

(1) $a^6\, b^1\, c^{-1}$ **(2)** $a^{t+2}\, b^3\, c^{3-r}$ **(3)** $a^{1/2}\, b^1\, c^{-9/2}$ **(4)** $a^{15}\, b^{5/2}\, c^{-35}$ **(5)** $t = \sqrt[3]{40}$ **(6)** $t = \log_7(200)$ **(7)** $t = \log_2(200/3)$ **(8)** $t = (\ln 4.32)/0.6$ **(9)** $t = 2^6$ **(10)** $t = e^{15}$ **(11)** $b = (8/3)^{1/4}$, $a = 30 \cdot (8/3)^{-5/4}$ **(12)** $b = (2.3/1.5)^{1/12}$, $a = (1.5^3)/(2.3^2)$.