Split "P" Soup
Locker Problems and Assorted Appetizers

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1. *Locker chaos.* A row of 100 lockers, all neatly closed, lines the side of a high school hallway. One Saturday, 100 students with mischief on their minds make their way into the hallway and start playing with the lockers in the following way. The first student goes by and opens every locker. The second student goes by and closes every *second* locker (starting with locker #2). The third student goes by and changes every *third* locker (starting with locker #3), opening it if it is closed and closing it if it is open. The remaining 97 students follow suit, the dth student changing every dth locker.

   After the students have all satisfied their entropic urges and have left to go bowling, which lockers are open?

2. *Rocks and lockers.* The next Saturday the lockers all stand open, except for the first locker which is closed. A smaller crowd of unrulies enters the school, each bearing a fist-size rock. The first student sees that the first open locker is locker #2, so she puts her rock inside that locker, and then closes that locker and every 2nd locker thereafter. The second student sees that now the first open locker is locker #3, so he puts his rock inside that locker, and then closes that locker and every 3rd locker thereafter, leaving closed the ones that are already closed. The remaining students follow suit, each one putting his/her rock in the first remaining open locker (call it locker #d), and then closes that locker and every dth locker thereafter (leaving closed the ones that are already closed). When they have all finished, each has put his/her rock in a locker and all of the lockers are closed.

   After these students have unburdened themselves in this way, which lockers have rocks in them? How many students were there?

3. *Counting change.* Here's a hint for the following pair of problems: between them, there are five possible solutions.

   (a) You have some pennies, dimes, and quarters—100 coins in all—totaling \$5.00. How many of each type of coin do you have?

   (b) You have some pennies, dimes, and quarters—100 coins in all—totaling \$4.99. How many of each type of coin do you have?

4. *Monkey business.* In this famous problem, n castaways are stuck on an island with a monkey. The castaways spent all of one day gathering a large pile of coconuts, which are to be split equally among themselves the following morning. In the middle of the night, one of the castaways gets up and quietly splits the pile of coconuts into n equal piles, leaving exactly one leftover coconut which he gives to the monkey. Being distrustful of the others, the castaway hides his nth share of coconuts, collects the remaining coconuts into a single pile, and goes back to sleep. A second castaway wakes up a little later and does the same thing: she splits the remaining coconuts in n equal piles with exactly one coconut left over for the monkey, hides her nth share, and collects the remaining coconuts together before going back to sleep. One by one, every other castaway does the same thing. The next day, the castaways split the pile of coconuts into n equal piles, finding that there is no leftover coconut for the monkey (who is oddly content with receiving no food). How many coconuts were in the original pile? (There is more than one solution, so for the sake of precision: what is the least number of coconuts that could have comprised the original pile?)