

Questions for Exploration
Escher Talk, Fields Institute
February, 2000

1. Geometry

- (a) What symmetries can you see in the different letters of the alphabet?
- (b) What is a reflection followed by a rotation if the centre of the rotation is on the reflection line? If it is not on the reflection line? What happens if the rotation is performed first and then the reflection?
- (c) Explore the composition of reflections, rotations, glide-reflections, and translations?

2. Linear Algebra (except for (b) which is Real Analysis)

- (a) Symbolically, what is an isometry of the plane?
- (b) If $f(x,y)$ is an isometry of the plane, what is $g(x,y) = f(x,y) - f(0,0)$? Why?
- (c) Do reflections, rotations, translations, and glide-reflections act linearly? (i.e. Are they affine transformations?)
- (d) If an isometry is a linear transformation, what type of matrix (or linear operator) represents the isometry?
- (e) Compare and contrast the eigenvalues and eigenvectors of reflections and rotations. Can you distinguish between the two types of isometries based on this information?
- (f) From the definition of an isometry of the plane (*an isometry is a mapping of the plane onto the plane that preserves distance*), delineate an argument that an isometry is affine.
- (g) Synthesize this material to convince yourself that there are only four types of isometries.

3 Symmetry Group of a Figure

- (a) What is the group operation? Why is it associative? What is the inverse of an isometry in this operation? What is the identity?
- (b) Are symmetry groups usually abelian?

4. Some Irrelevant Complex Analysis

- (a) If an analytic function is an isometry of the complex plane, what must it be?
- (b) Find an isometry of the unit disc, which is not an isometry of the complex plane?

5. Putting it all together.

Outline in writing the mathematical complexity of Escher's diagrams.