2021 MATH 1060H1F SYLLABUS Partial Differential Equations

http://www.math.toronto.edu/mccann/1060 or Quercus

Prof. Robert McCann mccann@math.utoronto.ca (3 day response time)
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Lectures in BA 6813 as of 28 Sept (reverting to AP 120 upon request)
Tuesday 14h10 – 16h00
Thursday 14h10–15h00
Office Hours: 16h15-17h00 in BA 6124

This course is a basic introduction to partial differential equations as they arise in physics, geometry and optimization. It is meant to be accessible to beginners with limited prior experience in the field (but possessing a firm grasp of multivariable calculus). It is also meant to introduce beautiful ideas and techniques which are part of most analysts' basic bag of tools. A key theme will be the development of techniques for defining and studying nonsmooth solutions to these equations, in which the nature of the nonsmoothness or its absence is the phenomenon of interest.

Required Text: Lawrence C. Evans: "Partial Differential Equations. 2nd Edition" GSM 19 Providence AMS 2010 Print ISBN 978-0-8218-4974-3 Electronic ISBN 978-1-4704-1144-2. Discounted to US $65 from US $99 at www.ams.org/bookstore/gsm-19-r

Topics to be covered: Chapters 1,2,3,5 and parts of 4, 6 and D.

A. Linear PDE (Chapter 2)

a) Second order elliptic partial differential operators. The Laplace operator. Harmonic functions. Maximum principle. The Dirichlet and Neumann problems. Existence, uniqueness and regularity. Harnack inequality. Green's functions.

b) The transport equation; the heat equation.

c) Hyperbolic partial differential equations. The wave equation. The Cauchy problem. Energy methods. Fundamental solutions. Domain of influence.

B. Introductions to nonlinear first-order equations (Chapter 3)

(a) Method of characteristics

(b) Hamilton-Jacobi equations

(c) Conservation laws in one dimension.

C. Sobolev spaces and their properties; weak solutions (Chapter 5)

D. 2nd order linear elliptic equations (Chapter 6, if time permits).

Grading Scheme (all assignments to be turned in on Crowdmark):

30% Weekly Assignments (accepted up to 5 days late, penalized 5% per day)
30% Project (accepted up to 10 days late, penalized 5% per day)
30% Test (14h10-17h00 Thursday 16 December)
10% Attendance+Participation or Test grade above, whichever is better

Project: The project will be to investigate a topic within partial differential equations or an application thereof which is not covered in-depth in this or your previous classes, and to summarize what you learn about it in a 6-8 page typed report, due Thursday November 25. (Ideally it would be a topic connected in some way or leading to an active area of research.) Please settle on a topic in consultation with me by October 14. Also tell me what sources from the literature you plan to use to research your topic (specific article(s) and/or book(s)) by Thursday October 14.

Rough Calendar:

Week 1 Introduction and Overview: Transport equation; Laplace's equation (harmonic functions, mean value formula, maximum principle, smoothness); Poisson's equation (fundamental solution)

Week 2 Representation formulas, estimates, analyticity, Harnack's inequality, Green's functions, energy methods (Dirichlet's principle)

Week 3 Heat equation: physical motivation, scaling, fundamental solution, maximum principle, Duhamel's principle,

Weak 4 mean value formula, smoothness, estimates, energy methods, uniqueness and backwards uniqueness

Week 5 Wave equation: motivation; solution in odd and even dimensions by spherical means (and descent), energy methods

Week 6 Nonlinear first-order equations: local solution by method of characteristics; global singularities

Week 7 Hamilton-Jacobi equations, Conservation laws, Hölder spaces

Week 8 Sobolev spaces: approximation, extension, traces

Week 9 Sobolev inequalities and embeddings; differentiability a.e.

Week 10 Second-order elliptic equations and boundary value problems; weak solutions (Lax-Milgram); energy estimates;

Week 11 compact operators and Fredholm theory; interior and boundary regularity (assuming C^1 coefficients)

Week 12 Moser Harnack inequality; DiGiorgi-Nash Hölder estimate

Academic Integrity

All suspected cases of academic dishonesty will be investigated following procedures outlined in the Code of Behaviour on Academic Matters. If you have questions or concerns about what constitutes appropriate academic behaviour or appropriate research and citation methods, please reach out to me. Note that you are expected to seek out additional information on academic integrity from me or from other institutional resources (for example, the University of Toronto website on Academic Integrity:

`http://academicintegrity.utoronto.ca/`

Equity, Diversity and Inclusion

The University of Toronto is committed to equity, human rights and respect for diversity. All members of the learning environment in this course should strive to create an atmosphere of mutual respect where all members of our community can express themselves, engage with each other, and respect one another's differences. U of T does not condone discrimination or harassment against any persons or communities.

Recordings of Lectures

I plan to record any lectures offered online, including your participation, on video and make it available to students in the course for viewing remotely and after each session. If you do not wish to appear in the recording, please switch off your camera before asking a question. Course videos and materials belong to your instructor, the University, and/or other sources depending on the specific facts of each situation and are protected by copyright. In this course, you are permitted to download session videos and materials for your own academic use, but you should not copy, share, or use them for any other purpose without the explicit permission of the instructor. For questions about the recording and use of videos in which you appear, please contact your instructor.

Accommodation of disabilities

The University provides academic accommodations for students with disabilities in accordance with the terms of the Ontario Human Rights Code. This occurs through a collaborative process that acknowledges a collective obligation to develop an accessible learning environment that both meets the needs of students and preserves the essential academic requirements of the University's courses and programs. Students with diverse learning styles and needs are welcome in this course. If you have a disability that may require

accommodations, please feel free to approach me and/or the Accessibility Services office, and register at `https://studentlife.utoronto.ca/department/accessibility-services/`

Religious accommodation

The University provides reasonable accommodation of the needs of students who observe religious holy days other than those already accommodated by ordinary scheduling and statutory holidays. Students have a responsibility to alert members of the teaching staff in a timely fashion to upcoming religious observances and anticipated absences and instructors will make every reasonable effort to avoid scheduling tests, examinations or other compulsory activities at these times. Please reach out to me as early as possible to communicate any anticipated absences related to religious observances, and to discuss any possible related implications for course work.

Accommodation of illness or other emergency

If the final assessment (test) is missed for a reason which the instructor deems to be legitimate, an oral or written make-up test will be administered at the instructor's sole discretion.

Family care responsibilities

The University of Toronto strives to provide a family-friendly environment. You may wish to inform me if you are a student with family responsibilities. If you are a student parent or have family responsibilities, you also may wish to visit the Family Care Office website at `familycare.utoronto.ca`

Additional sources

W Craig. A Course on Partial Differential Equations. Providence: AMS 2018 ISBN: 978-1-4704-4292-7

D Gilbarg and N Trudinger. Elliptic Partial Differential Equations of Second Order. 2nd Ed New York: Springer 1998 ISBN 3-540-13025-X

Jurgen Jost, Partial Differential Equations. 3rd Ed. New York: Springer, 2013. ISBN 978-1-4614-4808-2

R McOwen, Partial Differential Equations, (2nd ed), Hardcover: 2003 Prentice Hall ISBN 0-13-009335-1, Paperback: 2002 Pearson ISBN-13 978-0130093356