Mathematics 1901H Professor Robert McCann www.math.toronto.edu/1901.html
VARIATIONAL PROBLEMS IN PHYSICS, ECONOMICS, AND GEOMETRY

Lecture Hours: 9-12, 14-15, 15h30-17:30 on Sept 11-12, Oct 2-3 and Nov 6-7 in FI 230
Office Hours: Appointment in BA 6124 by email request

Offered in conjunction with the Fields' Institute Fall 2014 thematic semester of the same title, this unique graduate course will consist of a series of twelve 3 hour minicourses by distinguished local and visiting faculty on topics from the calculus of variations. Most of the lectures will be concentrated into two days each month. September 11-12 will be devoted to mathematical problems motivated by economics, such as matching models, mechanism design and applications. October 2-3 will be mathematical problems motivated by physics, including Gamma convergence techniques, phase-field models, coarsening, nonlinear dynamics and quantum many-body systems, Nov 6-7 will be devoted to problems in geometry and analysis, including inequalities, geometric control theory, curvature flows, and partial differential equations. Each two day graduate minischool is followed by a Fields Institute conference on related topics.

SYLLABUS

1. Mathematical transportation and its role in economic theory (Robert McCann, UofT Math)
2. Introduction to mechanism design (Xianwen Shi, UofT Economics)
3. Economic applications of matching models (Pierre-Andre Chiappori, Columbia Economics)
4. Applications of linear programming to economic theory (Rakesh Vohra, UPenn Business)
5. A crash course on Gamma convergence (Luigi Ambrosio, Scuola Normale Superiore di Pisa)
6. Variational methods for effective dynamics (Robert Jerrard, UofT Mathematics)
7. TBA (Felix Otto, Max-Planck Institute for Mathematics in the Natural Sciences at Leipzig)
8. The excitation spectrum for many-body quantum systems (Robert Seiringer, IST Austria)
9. Geometric stability problems for non-local functionals (Almut Burchard, UofT Math)
10. Mean curvature flow with surgery (Gerhard Huisken, University of Tübingen)
11. Geometric control and applications (Ludovic Rifford, University of Nice)
12. Monge-Ampère type equations (Neil Trudinger (ANU) or Young-Heon Kim (UBC))

GRADING SCHEME

20 % Lecture attendance and participation
5 % Identify chosen mentor, topic and 2 relevant references to Prof. McCann by Monday Oct 13
35 % Written project (due Monday Nov 24, ATTN Prof. McCann at Math Office BA 6290)
40 % Oral report (Friday Dec 5)

Late project submissions will be penalized by 5% per day late.

PROJECT GUIDELINES

Each student taking the course for credit will be required to complete a project related to an application of the calculus of variations (e.g. to physics, economics, geometry, analysis, image processing, design...) under the mentorship of one of the local faculty or semester-long visitors to the Fields Institute. Those who have indicated their willingness to mentor projects include Professors Almut Burchard, Wilfrid Gangbo, Young-Heon Kim, Robert Jerrard, and Robert McCann. Other choices are possible, subject to the agreement of the mentor and approval before Oct 13 by Professor McCann. Students are strongly encouraged to consult with Professor McCann about suitable topics, mentors and references well in advance of this date.

The project could have an original research component, but might more typically consist of the exposition of a result from the literature which is relevant to a topic of current research. A written report of 5-8 pages should be submitted to both the mentor and Professor McCann by Monday Nov 24. The student must also prepare a 20 minute oral introduction to their project, to be presented either to Professor McCann or the entire class on Friday Dec. 5 (or at any other mutually agreeable time before Dec 5).

ORAL REPORTS

The oral report will consist of two phases of approximately equal duration and weight, which must may be scheduled together or separately but must take place before Dec. 5. In the first (potentially public) phase the student will present a prepared 20 minute introduction to their project topic. In the second (private) phase, they will respond to questions about the content of their project and about a selection of the material covered in minicourse lectures during the term. To prepare for the second phase, for each 3 hour minicourse the student must write a short (one-page) synopsis summarizing the main points and 'take-home message'. They must bring their summary with them to the second phase of the oral report for the instructor to review. It is *strongly recommended* that the students complete these summaries within a few days of the time that the minicourse lectures are given, when the material is still fresh in their minds. It is instructor's hope that the process of preparing these summaries will help the student maximize the benefit derived from attending the minicourse lectures.