MATH 327H

Assignment 1 (From Henle §1 #9–12)

Due: Thursday Sept. 18

†9. Show that every polyhedron has either a triangular face or a trivalent vertex. Similarly, show that any polyhedron which does not contain a triangular face contains either a quadrilateral (4-gon) or a pentagon (5-gon). You may construct your argument by letting F_n be the number of n-gon faces, and V_n the number of vertices at which exactly n edges meet and verifying the following chain of equalities:

a) $F_3 + F_4 + F_5 + \ldots = F$ b) $V_3 + V_4 + V_5 + \ldots = V$

c) $3F_3 + 4F_4 + 5F_5 + \ldots = 2E$ d) $3V_3 + 4V_4 + 5V_5 + \ldots = 2E$

e) $(2V_3 + 2V_4 + 2V_5 + \ldots) - (F_3 + 2F_4 + 3F_5 + \ldots) = 4$

f) $(2F_3 + 2F_4 + 2F_5 + \ldots) - (V_3 + 2V_4 + 3V_5 + \ldots) = 4$

g) $(F_3 - F_5 - 2F_6 - 3F_7 - \ldots) + (V_3 - V_5 - 2V_6 - 3V_7 - \ldots) = 8$

h) $(3F_3 + 2F_4 + F_5 - F_7 - 2F_8 - \ldots) - (2V_4 + 4V_5 + 6V_6 + 8V_7 + \ldots) = 12$

†10. a) F and V play symmetrical roles in Euler's formula. Let two polyhedra be called *dual* if the number of faces of one is the number of vertices of the other and vice versa. What are the duals of the five Platonic solids?

b) Give a geometrical procedure for constructing a dual polyhedron to any given (say, convex if you like) polyhedron. What about a parallel procedure for dualizing planar cell complexes? What happens to the number of edges in each case?

†11. Find the solutions of the Diophantine equation $a^{-1} + b^{-1} = 2^{-1} + E^{-1}$ when a or b is allowed to equal 2. The corresponding polyhedra are *degenerate*. Draw them.

*12. Show that the Euler characteristic of the annulus is zero. Determine the Euler characteristic for the double annulus and the torus. What about the n-holed annulus?

*13. Consider a very small town consisting of N houses and three utility suppliers: the electricity, water, and gas companies.

a) For which integers N is it possible to run supply lines from each of the three utility companies to each of the N houses along the surface of the earth without any two supply lines crossing over each other? Use Euler's formula to prove your answer is correct.

BONUS: If the surface of the earth happened to be a torus (instead of a plane or a sphere) find the largest number of houses that can be serviced without any supply lines crossing. Prove it.

† Daggered problems may occasionally be selected for detailed grading at the TA's discretion, but more often will be subjected only to a cursory check to see whether a reasonable looking attempt to solve them appears to have been made.

* Starred problems will be graded in detail.