APM 426 / 1700 Assignment 11

Prof. McCann

Due: Thursday April 6*

*Those presenting projects on Thursday April 6 may turn in their assignments at our class meeting Friday April 7.

- 1. After a rocket ship passes through the Schwarzschild radius (horizon) of a black hole, show its radial co-ordinate must decrease at a rate given by $|dr/d\tau| \ge [2M/r-1]^{1/2}$ no matter how its engines are fired. Deduce that it reaches the singularity r = 0 in proper time $\tau \le \pi M$. Show that this upper bound is approached by a free falling (i.e., geodesic) motion starting from r = 2M with $E \to 0$. If the black hole has 10^6 solar masses, how many seconds could tick off a very small on board chronometer before it is torn apart by the huge gravitational forces at the singularity?
- 2. A radio commentator is describing his fall into a Schwarzschild black hole. Just before he crosses the Schwarzschild radius his broadcast frequency starts becoming enormously redshifted, with a time dependence exp(-t/constant), where t is the proper time of an observer at infinity. From the constant deduce the mass of the back hole.
- 3. Recall that the tidal force on a body is given by the geodesic deviation equation. A human being will be crushed when the acceleration gradient across its body reaches $400m/s^2$ (about 40g). Calculate the minimum mass of a black hoel which would permit the radio commentator to survive his fall along a radial geodesic long enough to reach the horizon of the black hole.
- I will be out of town Wednesday April 12, so am cancelling lecture that day and will hold my office hours 16h30-17h20 on Tuesday April 11 instead. Graded copies of this assignment, and the solution set, can be collected at my office hours.

To compensate, I have scheduled the following extra class meetings: 14h10-15h00 Friday March 31 in BA 3000 11h10-12h00 Friday April 7 in BA B-025

RECALL: TERM TEST 12h10-14h00 THURSDAY APRIL 13 in our usual classroom. You are encouraged to prepare two $8 \ 1/2 \ x \ 11$ sheets and bring them to the exam, with formulas written on both sides. No calculators or other aids will be permitted.

GOOD LUCK to all of you on your exams and in the future!