AARMS SUMMER COURSE 2011 – ANALYSIS AND GEOMETRY OF PDE'S

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Projects for student presentations (2nd part)

(1) **The moving planes method**.
- B. Gidas, W. Ni, L. Nirenberg, *Symmetry and related properties via the maximum principle*, Comm. Math. Physics 68 (1979), 209-243.

(2) **The convexity maximum principle method.**
- A. U. Kennington, *Power concavity and boundary value problems*, Indiana Univ. Math. J. 34 (1985), pp. 687–704.
- L. A. Caffarelli, J. Spruck, *Convexity properties of some classical variational problems*, Comm. in P. D.E. 7 (1982), pp. 1337-1379.
- N. Korevaar, *Convex solutions to nonlinear elliptic and parabolic boundary value problems*, Indiana Univ. Math. J. 32 (1983), pp. 603–614.

(3) **The Brascamp & Lieb method for proving log-concavity of the 1st eigenfunction of $-\Delta$ and a Brunn-Minkowski inequality for the corresponding eigenvalue.**
- H. J. Brascamp, E. Lieb, *On extension of the Brunn-Minkowski and Prékopa–Leindler inequality, including inequalities for log-concave functions, and with application to diffusion equation*, J. Funct. Anal. 22 (1976), pp. 366–389.

(4) **The constant rank method.**
- L. A. Caffarelli, A. Friedman, *Convexity of solutions to semilinear elliptic equations*, Duke Math. J. 52 (2) (1985), pp. 431–456.

(5) **The convex envelope method.**
- C. Bianchini, M. Longinetti, P. Salani, *Quasiconcave solutions to elliptic problems in convex rings*, Indiana Univ. Math. J. 58 (4) (2009), pp. 1565–1589.
- P. Salani, *Convexity of solutions and Brunn-Minkowski inequalities for Hessian equations in* $\mathbb{R}^3$, preprint (2011).

(6) **Brunn-Minkowski type inequalities for variational functionals**
- C. Borell, *Capacitary inequalities of the Brunn-Minkowski type*, Math. Ann. 263 (1983), pp. 179–194.
- P. Salani, *A Brunn-Minkowski inequality for the Monge-Ampre eigenvalue*, Adv. Math. 194 (2005), pp. 67–86.
- A. Colesanti, *Brunn–Minkowski inequalities for variational functionals and related problems*, Adv. Math. 194 (2005), pp. 105–140.

Additional references.

(1) O. Alvarez, J.-M. Lasry, P.-L. Lions, *Convex viscosity solutions and state constraints*, J. Math. Pures Appl. 76 (1997), pp. 265-288.

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(2) C. Borell, *Greenian potentials and concavity*, Math. Ann. 272 (1985), pp. 155–260.
(3) C. Borell, *Diffusion equation and geometric inequalities*, Potential Anal. 12 (2000), pp. 49–71.
(4) A. Colesanti, P. Salani, *Quasi–concave envelope of a function and convexity of level sets of solutions to elliptic equations*, Math. Nach. 258 (2003), pp. 3–15.
(5) B. Kawohl, *Rearrangements and convexity of level sets in PDE's*, Lecture Notes in Mathematics 1150, Springer–Verlag, Berlin, 1985.
(6) J. L. Lewis, *Capacitary functions in convex rings*, Arch. Rational Mech. Anal. 66 (1977), pp. 201–224.
(7) S. Sakaguchi, *Concavity properties of solutions to some degenerate quasilinear elliptic equations*, Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) 14 (1987), no. 3, pp. 403–421.