MAT 477H — Spring Semester — PRIMARY SOURCES

Weeks

0 16h10 Friday December 14 in BA 6183. Brief meeting to decide who presents what. E-mail your preferences to `mccann@math.toronto.edu` by December 12 in case you are unable to attend. March 3-11 absence.

1 Introduction to statistical mechanics, the mathematics of phase transitions [1] [2] [3], including the Ising model in one-dimension.

2 R.B. Griffiths. A proof that the free energy of a spin system is extensive. *J. Math. Phys.* **5** (1964) 1215–1222.

A warm-up to explore ideas related to existence of the 'thermodynamic' (i.e., large system size) limit and the role of convexity.

3 H.A. Kramers and G.H. Wannier. Statistics of the two-dimensional ferromagnet. Part I. *Phys. Rev.* **60** (1941) 252–262.

Especially §4, where a duality was discovered which predicts the temperature at which a single phase transition in the 2-d Ising model would have to occur. This key paper also introduced the transfer matrix.

4 R. Peierls. On Ising's model of ferromagnetism. *Proc. Cambridge Phil. Soc.* **32** (1936) 477–481 and R.B. Griffiths. Peierls proof of spontaneous magnetization in a two-dimensional Ising ferromagnet. *Phys. Rev.* **136** (1964) A437–A439.

After solving his toy model for magnetism explicitly in one-dimension, Ising incorrectly concluded his model predicted no spontaneous magnetization (or long range order) in any dimension, and thus could not address phase transitions. Peierls (and Griffiths) disproved this claim, by demonstrating symmetry breaking (non-uniqueness of Gibbs state) at low temperature in two dimensions. Although the 2-d Ising model was subsequently solved explicitly, the Peierls contour argument remains a crucial estimate for statistical problems defying exact analysis.

5 R.B. Potts. Some generalized order-disorder transformations. *Proc. Cambridge Phil. Soc.* **48** (1958) 106–109.

A generalization to systems with more than two possible states at each site (red, green, blue instead of black and white).

6 E.H. Lieb. Exact Solution of the Problem of the Entropy of Two-Dimensional Ice. *Phys. Rev. Lett.* **18** (1967) 692–694.

Exploits transfer matrix methods to count orientational degrees of freedom for water molecules held in a square lattice by hydrogen bounds. Though the square lattice does not capture the real three-dimensional structure of ice [5], this paper was a technical and conceptual triumph in accounting for the correlations which influence this experimentally measured quantity.

7 M. Kac and J.C. Ward. A combinatorial solution of the two-dimensional Ising model. *Phys. Rev.* **88** (1952) 1332–1337.

Structural connections of the Ising model with determinants.

8 P.W. Kasteleyn. The statistics of dimers on a lattice. *Physica* **27** (1961) 1209–1225.

Counting domino tilings of a lattice using determinants and Pfaffians.

9 C.A. Hurst and H.S. Green. New solution of the Ising problem for a rectangular lattice. *J. Chem. Phys.* **33** (1960) 1059–1062.

The 2-d Ising model's partition function in all its glory, using combinatorial methods of 7–8 much simpler than Onsager's original solution.

10 I.M.H. Etherington and A. Erdélyi and I.M.H. Etherington. Some problems of non-associative combinations I and II. *Edinburgh Mathematical Notes* **32** (1940) 1–12.

Geometrical combinatorics using generating functions.

11 F.J. Dyson. The dynamics of a disordered linear chain. *Phys. Rev.* **92** (1953) 1331-1338.

A classic! Calculates the average distribution of vibrational frequencies for a long chain of randomly coupled masses and springs.

MAT 477H: Spring Semester Supplementary

References

[1] C.J. Thompson. *Mathematical Statistical Mechanics.* Princeton University Press, Princeton, 1972.

[2] D. Ruelle. *Statistical Mechanics: Rigorous Results.* Addison-Wesley, New York, 1969.

[3] G.L. Sewell *Quantum mechanics and its emergent macrophysics.* Princeton University Press, Princeton, NJ, 2002.

[4] R.B. Griffiths. Correlations in Ising ferromagnets. I. *J. Math. Phys.* **8** (1967) 478–483; II. External magnetic fields *J. Math. Phys.* **8** (1967) 484–489; III. A mean-field bound for binary correlations *Comm. Math. Phys.* **6** (1967) 121–127.

[5] L. Pauling. The structure and entropy of ice and other crystals with some randomness of atomic arrangement. *J. Amer. Chem. Soc.* **37** (1935) 2680–2684.

[6] P.W. Kasteleyn. The statistics of dimers on a lattice. *Physica* **27** (1961) 1209–1225.