University of Toronto
Midterm 1 - Oct 10, 2012

MAT 244F – Ordinary Differential Equations
Instructors: E. Meinrenken & T. Bazett
Duration – 50 mins

NO CALCULATORS, CELL PHONES OR OTHER AIDS ALLOWED.

Total Marks=40

Problem #1: [6 points] Sketch several isoclines, the direction field, and integral curves for the following equation. Please draw as neatly as possible and clearly identify which curves are isoclines and which are solutions.

$$y' = y^2 + t^2 - 1$$

Problem #2: [24 points, 6 points each] State whether the equation is linear, separable, homogeneous, exact, or none of these. Then, solve the equation.

a)

$$(3t^2 - 2ty + 2) + (6y^2 - t^2 + 3)y' = 0$$

b)

$$y' = \frac{t^2 + 5ty + 4y^2}{t^2}, \quad t > 0$$

c)

$$y' = 2(e^t - y)$$

d)

$$\frac{\partial y}{\partial x} = \frac{x^2 - 1}{y^2 + 1}, \quad y(0) = 1$$

Problem #3: [10 points total] Radioactive Barium (Ba^{140}) decays into Lanthanum, while Lanthanum is again radioactive and decays into Cerium. Let $Q_1(t)$ denote the amount of Barium, $Q_2(t)$ the amount of Lanthanum, and $Q_3(t)$ the amount of Cerium at time t.

Let r_1 be the decay rate of Barium, and r_2 the decay rate of Lanthanum. Thus

$$Q_1'(t) = -r_1 Q_1(t).$$

a) [3 points] Find $Q_1(t)$, for an initial condition $Q_1(0) = K$ (a positive constant).

b) [2 points] Write a differential equation in $Q_2(t)$, using your answer from part a).

c) [5 points] Solve the differential equation from part b), for an initital condition $Q_2(0) = 0$.