Exercise 1. Compact n-manifolds M_0, M_1 are said to be *cobordant* when there is a compact $n+1$-manifold N with boundary such that $\partial N = M_0 \sqcup M_1$. Show that if M_1 is obtained from M_0 by a surgery (on some embedded sphere, as defined in class), then they are cobordant. Draw examples of such cobordisms for $n = 0, 1, 2$.

Exercise 2. Identify the manifold obtained from ± 1 surgery on the unknot in S^3.

Exercise 3. Let Σ_g be the orientable Riemann surface of genus g. Express the 3-manifold $S^1 \times \Sigma_g$ as a sequence of handle attachments on the 3-ball.

Exercise 4. Let M be a compact n-manifold admitting a Morse function with only two critical points. Prove that M must be *homeomorphic* to S^n. Hint: Use Theorem A, and beware that M need not be *diffeomorphic* to the n-sphere: in fact this result is usually used to prove that the exotic smooth spheres are indeed homeomorphic to spheres.

Exercise 5 (Bonus – difficult but worth thinking about)**.** Give a nice proof of the result of Cerf which states that if $F : M \times [0,1] \longrightarrow \mathbb{R}$ is a smooth function such that $F(\cdot, 0)$ and $F(\cdot, 1)$ are Morse functions on M, then F can be approximated arbitrarily well in the C^∞ topology by a function $\tilde{F}$ such that $\tilde{F}(\cdot, t)$ is a Morse function on M for all but finitely many values of t, at which $F(\cdot, t)$ has only one degenerate critical point, and such that there exists a coordinate system originating at this point where

$$F(x, t) = c + x_1^3 + \epsilon_1 t x_1 + \epsilon_2 x_2^2 + \cdots + \epsilon_n x_n^2,$$

where $\epsilon_i \in \{\pm 1\}$.