Sets

A **set** is a collection of distinct objects, viewed as an object in its own right. If an object x is contained in the set S, we say

"x is an element of S"

or $x \in S$

Ex.: • $P = \{R, G, B\}$ a set of three letters

• $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \cdots\}$ the set of integers

• $\mathbb{Q} = \{p/q : p \in \mathbb{Z},\ q \in \mathbb{Z} \setminus \{0\}\}$ the rational numbers

• $\mathbb{R}$, $\mathbb{C}$ the real and complex numbers

• ϕ the empty set.

A set S is **finite** when it has a finite number of elements. This number,

$$|S| \in \{0, 1, 2, \cdots\},$$

is called the **cardinality** of S.

Ex.: $|\phi| = 0$
$|P| = 3$

Maps between sets

the DOMAIN of f the CODOMAIN of f.

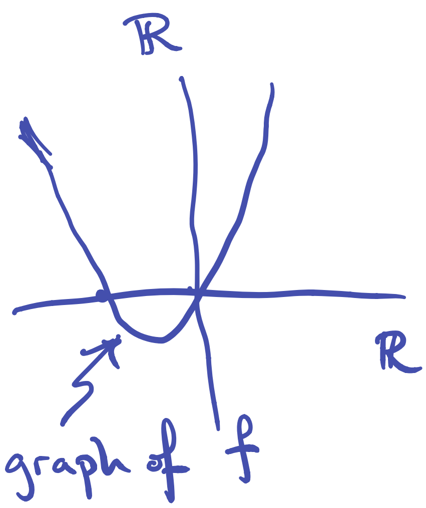
A **map** f from the set X to the set Y assigns to each $x \in X$ a unique element $f(x) \in Y$.

In symbols
$$X \xrightarrow{f} Y$$
$$x \longmapsto f(x)$$

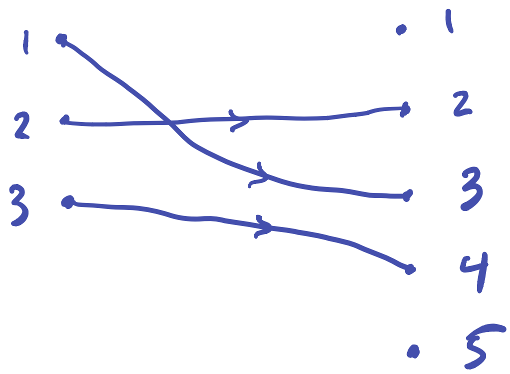
Ex.: • A RGB colour is a map $P \to \{0, 1, \dots, 255\}$.

e.g. white : $R \mapsto 255$
$G \mapsto 255$
$B \mapsto 255$

• You have studied maps $\mathbb{R} \xrightarrow{f} \mathbb{R}$ such as $f(x) = x^2 + x$



• a map from $\{1, 2, 3\}$ to $\{1, 2, 3, 4, 5\}$ may be drawn as follows:

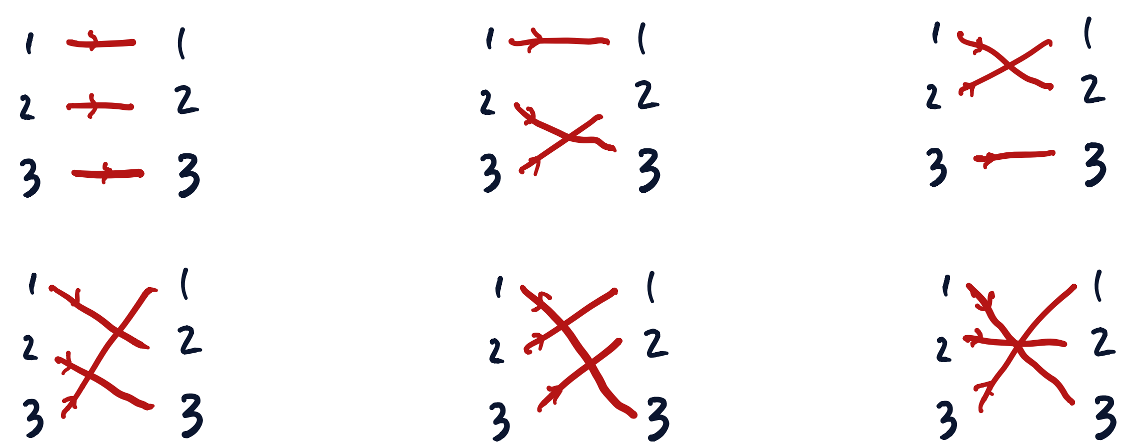


f: $1 \mapsto 3$
$2 \mapsto 2$
$3 \mapsto 4$

The map $f : X \to Y$ is

- <u>Injective</u> when different inputs give different outputs
  i.e. $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.
- <u>Surjective</u> when all possible outputs are achieved
  i.e. $\forall y \in Y \ \exists x \in X : f(x) = y$.
- <u>Bijective</u> when both injective & surjective.

<u>Ex.:</u> Bijections $\{1,2,3\} \to \{1,2,3\}$.



<u>Ex.:</u> Any set X has an identity map to itself

$\mathrm{Id}_X : X \to X$ which takes $x \in X$ to x

for all $x \in X$. It is a bijection.

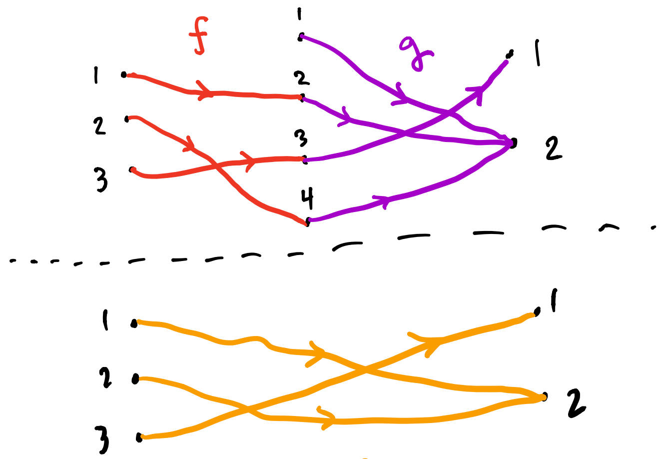
<u>Q.:</u> How many bijections from X to X are there, if $|X| = n$?

Composition

The **composition** of maps $f : X \to Y$ and $g : Y \to Z$ is the map taking $x \in X$ to $g(f(x)) \in Z$ and is denoted by $g \circ f$.

$$\begin{tikzcd} X \arrow[r, "f"] \arrow[rr, bend right, "g \circ f"'] & Y \arrow[r, "g"] & Z \end{tikzcd}$$

Ex.:



$g \circ f$

Q: Prove that composition is associative, i.e. for maps $S \xrightarrow{f} T \xrightarrow{g} U \xrightarrow{h} V$

show $h \circ (g \circ f) = (h \circ g) \circ f$