This is a preview of what students will see when they are submitting the assignment. Interactive features are disabled.

Assignment 1

Due: Monday September 21, 2020 11:59 PM (Eastern Daylight Time)

Assignment description

Remember that uploading your assignment will take time. That is why I set the deadline at midnight. Late assignments are not accepted.

The first assignment is only be based on the material presented in the first two classes. I strongly recommend that you **complete this assignment on your own**, with no assistance from classmates. Any questions, no matter how basic, should be asked on the Q&A forum. Just avoid asking questions which are too close to the actual assignment question.

Some do's and dont's when writing proofs: We are ok with simple logic symbols such as ∀, ∃, ⇒, ⇐, ⇔, provided that you use them properly. In case of doubt, plain words are often better. Please do not use the ∴ symbol since this is very uncommon in mathematical writing. Also, some other logic symbols such as ∧ and ∨ are not commonly used in math texts (outside of mathematical logic), it's better to use words. Finally, avoid the use of × for multiplications (of numbers etc) since there are too many other meanings attached to this symbol -- the cross product of vectors, the variable x, and so on. (Use · instead.)

Finally, it is your responsibility to ensure that your written work is legible and neat. The TAs are fully within their right to remove points based on bad presentation which makes your work illegible.

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (2 points)

Let X be a finite set of cardinality n.

a) What is the cardinality of the set $\mathrm{Map}(X, X)$ of maps from X to itself?

b) What is the cardinality of the set of bijections from X to itself?

Give clean and clear arguments for both answers.

Q2 (1 point)

Let $f : S \to T$, $g : T \to U$, and $h : U \to V$ be maps. Prove that composition is associative, i.e.

$$h \circ (g \circ f) = (h \circ g) \circ f$$

Q3 (5 points)

Let $f : X \to Y$ be a map. The map $g : Y \to X$ is called an *inverse* of f when it satisfies the properties

$$g \circ f = I_X \qquad \text{and} \qquad f \circ g = I_Y,$$

where I_Z means the identity map from Z to itself.

a) Prove that if an inverse of the map f exists, then it is unique.

b) Give an example of a non-invertible map f.

c) Prove that f is invertible (i.e. it has an inverse) if and only if it is a bijection.

d) Suppose $f : X \to Y$ and $g : Y \to X$ are such that $g \circ f = I_X$. Does it follow that $f \circ g = I_Y$? Why?

e) Let f, g be as in part d), with the additional assumption that f is surjective. Does it follow that $f \circ g = I_Y$? Why?

Q4 (2 points)

Let $f : X \to X$ be a map of sets. A *fixed point* of f is an element $x \in X$ such that $f(x) = x$.

a) For each bijection from $\{1, 2, 3\}$ to itself, determine the fixed points.

b) Let M be the set of bijections from $\{1, 2, 3\}$ to itself. Sending a map to its inverse defines a bijection $i : M \to M$. Determine the fixed points of i.

Q6 (1 point)

The *power set* of a set X is the set consisting of all subsets of X. The power set of X is usually written $\mathcal{P}(X)$. If X is a set with n elements (i.e. with cardinality n), what is the cardinality of $\mathcal{P}(X)$?

Q5 (1 point)

A *partition* of a set X is a decomposition of X into a disjoint union of nonempty subsets. More precisely, a partition P is a subset of $\mathcal{P}(X)$, not containing the empty set, whose elements satisfy the following conditions:

a) If Y, Y' are elements of P then $Y \cap Y' = \emptyset$, and

b) $\bigcup_{Y \in P} Y = X$.

How many partitions are there of a set of cardinality 6? List all the partitions of the set $\{1, 2, 3, 4, 5, 6\}$.