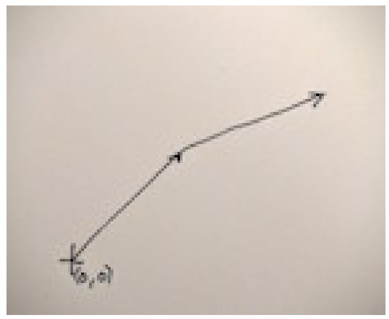
Exercise 1. Prove that, using the definition of submanifold given in class, any submanifold $L \subset M$ inherits a smooth structure from M. Also show that using this smooth structure, the inclusion map $i : L \to M$ is an embedding.

Exercise 2. Prove Proposition 3.8 in the notes:

Proposition 3.8 Let K be a manifold with boundary where L, M are without boundary. Assume that $f : K \to M$ and $g : L \to M$ are smooth maps such that both f and ∂f are transverse to g. Then the fiber product $K \times_M L$ is a manifold with boundary equal to $\partial K \times_M L$.



Exercise 3. Consider an planar arm, consisting of two rigid unit lengths joined with a hinge at endpoints. Assume that the shoulder is fixed at the origin, as shown above.

1. What is the space of configurations X of this planar arm?

2. Suppose we fix the extremity $p \in \mathbb{R}^2$ of the arm. What is the configuration space then? How does this depend on the particular placement of p?

3. Consider the map $\pi : X \to \mathbb{R}^2$ given by p, the position of the extremity. Prove that it is a smooth map and determine the critical points and critical values of this map. Describe the manifold $\pi^{-1}(p)$ for p lying in each connected component of the set of regular values.

4. Now let X be the configuration space for a planar arm made of three segments rather than the above two; repeat the analysis in question 3.

5. Finally, let X be the configuration space for a planar arm made of four segments; repeat the analysis in question 3.

Bonus 3.1. Let K, L be submanifolds of a manifold M, and suppose that their intersection $K \cap L$ is also a submanifold. Then K, L are said to have *clean* intersection when, for each $p \in K \cap L$, we have $T_p(K \cap L) = T_pK \cap T_pL$. Show that there are coordinates near $p \in K \cap L$ such that K, L, and $K \cap L$ are given by linear subspaces of $\mathbb{R}^n$ of the form $V(x^{i_1}, \ldots, x^{i_k})$ for some subset of the coordinates. It is useful to use the algebraic geometry notation $V(x^1, \ldots, x^k)$ to mean the "vanishing" subspace $x^1 = \cdots = x^k = 0$.