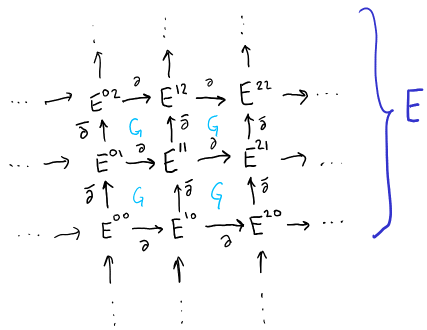
Reading: Hatcher §2.2.

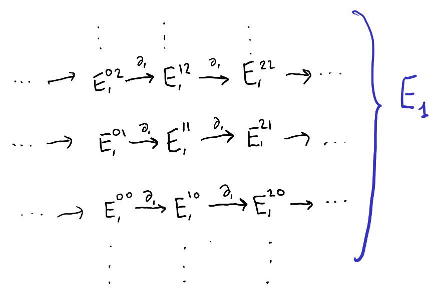
Exercise 1. Hatcher, Section 2.2 Exercises 8, 10, 20, 21, 22.

Exercise 2. Let E be a double complex, meaning that E is a $\mathbb{Z} \times \mathbb{Z}$ graded vector space,

$$E = \bigoplus_{(i,j) \in \mathbb{Z} \times \mathbb{Z}} E^{i,j},$$

equipped with two operators $\partial, \bar{\partial}$ of degrees $(1,0)$ and $(0,1)$ respectively, such that $\partial^2 = \bar{\partial}^2 = 0$ and $\partial\bar{\partial} = \bar{\partial}\partial$.





1. Produce from E another $\mathbb{Z} \times \mathbb{Z}$ graded vector space E_1, by taking $\bar{\partial}$ cohomology:

$$E_1^{i,j} = \frac{\ker \bar{\partial}|_{E^{i,j}}}{\bar{\partial}(E^{i,j-1})}.$$

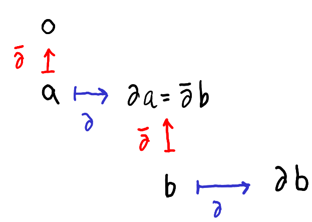
Prove that both $\partial, \bar{\partial}$ induce maps $\partial_1, \bar{\partial}_1$ of degree $(1,0), (0,1)$ respectively on E_1, that $\bar{\partial}_1 = 0$, and that $\partial_1^2 = 0$.

2. Produce from E_1 another $\mathbb{Z} \times \mathbb{Z}$ graded vector space E_2, by taking ∂_1 cohomology:

$$E_2^{i,j} = \frac{\ker \partial_1|_{E_1^{i,j}}}{\partial_1(E_1^{i-1,j})}.$$

As before, the map induced by ∂_1 is zero, but prove that the following procedure defines a well-defined map $\partial_2 : E_2 \to E_2$ of degree $(2,-1)$:

Let $a \in E^{i,j}$ be such that $\bar{\partial}a = 0$ and $\partial a = \bar{\partial}b$. Then $\partial b \in E^{i+2,j-1}$.



Finish by proving that $\partial_2^2 = 0$.