18.969 Topics in Geometry, MIT Fall term, 2006

Graduate course in generalized geometry

Lecturer: Marco Gualtieri
Time: TR 11-12:30
Room: 2-255

This will be an introductory course in generalized geometry, with a special emphasis on Dirac geometry, as developed by Courant, Weinstein, and Ševera [1],[7], as well as generalized complex geometry, as introduced by Hitchin [4]. Dirac geometry is based on the idea of unifying the geometry of a Poisson structure with that of a closed 2-form, whereas generalized complex geometry unifies complex and symplectic geometry. For this reason, the latter is intimately related to the ideas of mirror symmetry. The basic reference will be [3], but we will also draw from more recent developments in the physics literature, e.g. [5],[2],[6],[8], among others.

A basic familiarity with complex and symplectic manifolds will be assumed; here is a list of topics which will be covered in the lecture course:

- Gerbes, B-fields, and exact Courant algebroids;
- Relation to sigma models in physics and baby String theory;
- Linear algebra of a split-signature real bilinear form; pure spinors;
- Generalized Riemannian structures and the generalized Hodge star;
- Integrability, Dirac structures, Lie algebroids and bialgebroids;
- Generalized complex structures; examples of such;
- Generalized holomorphic bundles; the Picard group;
- Kodaira-Spencer-Kuranishi deformation theory for generalized complex structures;
- Local structure theory for generalized complex structures;
- Generalized Kähler geometry;
- Hodge decomposition theorem for Generalized Kähler structures;
- Hermitian geometry; the Gray-Hervella classification
- Equivalence theorem Generalized Kähler=Bihermitian
- Generalized Calabi-Yau structures and the Hitchin functional
- Ramond-Ramond versus Neveu-Schwarz fluxes; D-branes.

References

[1] Courant, T. *Dirac manifolds.* Trans. Amer. Math. Soc., **319**, 631–661, 1990.

[2] Graña, M., Minasian, R. , Petrini, M., and Tomasiello, A. *Generalized structures and N=1 vacua.* hep-th/0505212.

[3] Gualtieri, M. *Generalized complex geometry.* Oxford D. Phil. thesis, math.DG/0401221.

[4] Hitchin, N. *Generalized Calabi-Yau manifolds.* Q. J. Math. **54**, 281 – 308, 2003. math.DG/0209099.

[5] Kapustin,A. and Li, Y. *Topological sigma-models with H-flux and twisted generalized complex manifolds.* hep-th/0407249.

[6] Lindstrom, U., Roček, M., von Unge, R., and Zabzine, M. *Generalized Kähler manifolds and off-shell supersymmetry.* hep-th/0512164.

[7] Ševera, P. and Weinstein, A. *Poisson geometry with a 3-form background.* Prog. Theo. Phys. Suppl. **144**, 145 – 154, 2001. math.SG/0107133.

[8] Zucchini, R., *Generalized complex geometry, generalized branes and the Hitchin sigma model*, hep-th/0501062.