APM 236   Second Midterm 3/20/2002   50 points possible

You may not use calculators, cell phones, or PDAs during the exam. Partial credit is possible. Please read the entire test over before starting. Please put a box around your solutions so that the grader can find them easily.

Print your name clearly:

Print your student number clearly:

Please sign here:

Problem 1

Problem 2

Problem 3

Problem 4

Problem 5

Problem 6

Total

1. (5 points) Consider the following linear programming problem:

$$\text{Maximize } 10x_1 + 3x_2$$
$$\text{Subject to}$$

$$
\begin{array}{rrrcr}
x_1 & +2x_2 & +3x_3 & \leq & 1 \\
 & 4x_2 & & \leq & 2 \\
-x_1 & & +9x_3 & \leq & 0 \\
x_1 & -x_2 & & \leq & -4 \\
\end{array}
$$

where $x_1, x_2, x_3 \geq 0$.

What is the dual problem?

2. (10 points) Consider the following linear programming problem:

$$\text{Maximize } x_2 - x_3 + 3x_4$$
$$\text{Subject to}$$

$$
\begin{array}{rrrrcr}
3x_1 & -x_2 & & +2x_4 & \leq & -10 \\
6x_1 & +x_2 & +x_3 & +2x_4 & \leq & 0
\end{array}
$$

$$\text{where } x_1, x_2, x_3, x_4 \geq 0.$$

You want to use the simplex method to find an optimal solution. Give the first two tableaux.

more room to work on back!

3. (10 points) Consider the following linear programming problem:

$$\text{Maximize } x_1 + x_2$$
$$\text{Subject to}$$

$$
\begin{array}{rrcl}
-x_1 & +x_2 & \leq & 1 \\
4x_1 & -x_2 & \leq & 8
\end{array}
$$

where $x_1, x_2 \geq 0$.

At one point during the simplex method, the basic variables are x_2 and x_4. Give the tableau at that time.

more room to work on back!

4. (10 points) Give an example of a linear programming problem whose dual has feasible solutions but no finite optimal solution.

5. (5 points) Complete the following tableau:

c_B		x_1	x_2	x_3	x_4	x_B
		1	?	1	?	
?	?	1	0	$-3/4$	0	$5/2$
2	?	0	0	?	1	$15/2$
0	?	0	1	0	0	0
		?	?	$3/4$	?	?

6. Consider the following tableau:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	
x_3	0	0	1	0	0	-2	0	2	0
x_5	0	-2	0	10	1	2	0	0	1
x_7	0	-1	0	10	0	4	1	-5	1
x_1	1	2	0	-3	0	-4	0	1	0
	0	-4	0	-2	0	-1	0	-10	10

a. (1 pt) There are four possible choices of incoming variables. What are they?

b. (6 pt) For each possible incoming variable, give the outgoing variable(s) that are possible. (Think something like, "If x_1 is incoming then I have to look at x_3, x_2, x_5, and x_8. I know that x_3 and x_2 are not going to be outgoing because... And x_5 won't be outgoing because... So for x_1 the only possible outgoing variable is...")

c. (3 pt) You now have a collection of pairs of incoming and outgoing variables. List them. Now, discuss which pair you would choose and why you'd choose it.