APM 236 Second Midterm March 19, 2003 100 points possible

You may not use calculators, cell phones, or PDAs during the exam. Partial credit is possible. Please read the entire test over before starting. Please put a box around your solutions so that the grader can find them easily.

Print your name clearly:

Print your student number clearly:

Please sign here:

Problem 1	out of 10
Problem 2	out of 15
Problem 3	out of 20
Problem 4	out of 15
Problem 5	out of 10
Problem 6	out of 15
Problem 7	out of 15
Total	out of 100

1. (10 pt) Use the simplex method to solve the following linear programming problem:

$$\text{Maximize } 2x_1 - x_2$$
$$\text{subject to}$$

$$\begin{aligned} 2x_1 \quad +x_2 &\leq 4 \\ x_1 \qquad\quad &\leq 2 \end{aligned}$$

$$\text{where } x_1, x_2 \geq 0.$$

2. (15 pt) Use the simplex method to solve the following linear programming problem:

$$\text{Maximize } \tfrac{3}{2}x_2 + \tfrac{1}{2}x_3$$
$$\text{subject to}$$

$$\begin{array}{lllll} & -\frac{1}{2}x_2 & +\frac{1}{2}x_3 & +x_4 & = 2 \\ x_1 & & -x_3 & & = 2 \end{array}$$

$$\text{Where } x_1, x_2, x_3, x_4 \geq 0$$

3. (20 pt) Use the simplex method to solve the following linear programming problem:

$$\text{Minimize } -2x_1 + 2x_2$$
$$\text{subject to}$$

$$\begin{aligned} 2x_1 \quad -x_2 &\leq -3 \\ 4x_1 \quad +2x_2 &\leq 2 \end{aligned}$$

$$\text{Where } x_1, x_2 \geq 0.$$

4. (15 pt) Consider the linear programming problem:

$$\text{Maximize } (3\ 1\ 2\ 4)^T\ \vec{x}$$
$$\text{subject to}$$
$$\begin{pmatrix} -1 & 3 & 1 & 1 \\ 2 & 2 & -2 & 1 \end{pmatrix} \vec{x} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$
$$\text{where } \vec{x} \geq \vec{0}.$$

You are told that at some point while using the simplex method to solve this problem, the basic variables are x_2 and x_4. Find the simplex tableau at that time. *Do not solve this problem by starting the simplex method from scratch and pivoting until you have basic variables x_2 and x_4.*

5. (10 pt) Consider the following tableau:

	x_1	x_2	x_3	x_4	x_5	x_6	
x_1	1	0	-1	2	1	9	10
x_2	0	1	1	1	0	1	2
	0	0	-3	2	-1	3	11

a) You arrived at this tableau while applying the simplex method to solve a linear programming problem in which an objective function is to be **maximized**. What choice of entering and departing variable should you now take if you want the objective function to **increase** as much as possible? How much will the objective function increase if you make this choice?

b) You arrived at this tableau while applying the simplex method to solve a linear programming problem in which an objective function is to be **minimized**. What choice of entering and departing variable should you now take if you want the objective function to **decrease** as much as possible? How much will the objective function decrease if you make this choice?

6. (15 pt) Consider the linear programming problem:

$$\text{Minimize } x_1 + x_2$$
$$\text{subject to}$$

$$\begin{array}{rrl} x_1 & +x_2 & \geq -1 \\ -x_1 & +x_2 & \leq 3 \end{array}$$

$$\text{where } x_2 \geq 0$$

a) What is the dual linear programming problem of the above problem?

b) The optimal solution of the primal problem is at $(x_1, x_2) = (-2, 1)$. Use complementary slackness to find an optimal solution of the dual problem.

7. (15 pt) Consider the (primal) linear programming problem:

$$\text{Maximize } \vec{c}^T \vec{x}$$
$$\text{subject to}$$
$$A\vec{x} \le \vec{b},$$
$$\text{and } \vec{x} \ge \vec{0},$$

and its dual linear programming problem:

$$\text{Minimize } \vec{b}^T \vec{w}$$
$$\text{subject to}$$
$$A^T \vec{w} \ge \vec{c},$$
$$\text{and } \vec{w} \ge \vec{0}.$$

a) Prove that if $\vec{x_0}$ is a feasible solution of the primal problem and $\vec{w_0}$ is a feasible solution of the dual problem then

$$\vec{c}^T \vec{x_0} \le \vec{b}^T \vec{w_0}.$$

b) Prove that if $\vec{x_0}$ is a feasible solution of the primal problem and $\vec{w_0}$ is a feasible solution of the dual problem such that

$$\vec{c}^T \vec{x_0} = \vec{b}^T \vec{w_0}$$

then $\vec{x_0}$ is an **optimal** solution of the primal problem.