Homework #2, due October 8 at the beginning of class.

Can changing coordinates change the type of the PDE? Consider the second-order PDE

$$a(x,y)\,u_{xx}(x,y) + b(x,y)\,u_{xy}(x,y) + c(x,y)\,u_{yy}(x,y) = d(x,y,u(x,y),u_x(x,y),u_y(x,y)).$$

Consider the change of coordinates

$$\mu = \mu(x,y) \qquad \eta = \eta(x,y).$$

Define the function v via

$$v(\mu(x,y),\eta(x,y)) = u(x,y).$$

What PDE does v satisfy? Is it possible that the PDE for u and the PDE for v are of different types? (I.e. can you change a PDE from hyperbolic to elliptic by changing coordinates?)

Can flattening the boundary change the type of the PDE? Consider the second-order PDE

$$a(x,y)\,u_{xx}(x,y) + b(x,y)\,u_{xy}(x,y) + c(x,y)\,u_{yy}(x,y) = d(x,y,u(x,y),u_x(x,y),u_y(x,y)).$$

Consider the change of coordinates

$$\mu = x \qquad \eta = y - \gamma(x).$$

First, figure out what this change of coordinate means for three cases: $\gamma(x) = mx$ for some nonzero m, $\gamma(x) = x^2$, $\gamma(x) = \sin(x)$. (For example, on the xy plane you can draw curves corresponding to constant values of μ and constant values of η, resulting in "distorted graph paper".) Define the function v via

$$v(\mu(x,y),\eta(x,y)) = u(x,y).$$

What PDE does v satisfy? Is it possible that the PDE for u and the PDE for v are of different types? *Obviously, this problem is a special case of the previous problem. And so the answer to this problem should be pretty short and sweet.*

Classify the PDEs and reduce them to standard form:

$$u_{xx} - 2u_{xy} + u_{yy} + 3u_x - u_y = 0.$$

$$u_{xx} - 2\cos(x)\,u_{xy} + (2 - \sin^2(x))\,u_{yy} + u = 0.$$

Section 2.1: Please do problems 1, 5, and 7.

Section 2.2: Please do problems 4, 5, and 9.