Extra Questions for Chapter 2 Material

Nicholas Hoell

March 9, 2017

1. How many 1×1 elementary matrices are there?
2. How many nonzero entries can an $n \times n$ elementary matrix have?
3. Is the $n \times n$ zero matrix elementary in each n? If not, is there *any* n for which it is elementary?
4. Is the product of two elementary matrices always elementary? If not, is it *ever* elementary?
5. Suppose that AB is invertible. Is A? If so prove it. If not, give a counterexample.
6. Define $S : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ via $S(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T A \mathbf{y}$ where A is an $n \times n$ symmetric matrix.
   (a) Find a choice of A such that $\mathbf{x} \cdot \mathbf{y} = S(\mathbf{x}, \mathbf{y})$.
   (b) Show that, for each *fixed* $\mathbf{x}$, the map $\mathbf{y} \mapsto S(\mathbf{x}, \mathbf{y})$ is linear.
   (c) Show that $S(\mathbf{x}, \mathbf{y}) = S(\mathbf{y}, \mathbf{x})$ (this is not as easy as you might think, be careful)
   (d) Show that $S(c\mathbf{x}, \mathbf{y}) = S(\mathbf{x}, c\mathbf{y}) = cS(\mathbf{x}, \mathbf{y})$.
7. Define $T : \mathbb{R}^n \to \mathbb{R}$ via $T(\mathbf{x}) = \mathbf{x}^T A \mathbf{x}$ where A is an $n \times n$ symmetric matrix. Notice, $T(\mathbf{x}) = S(\mathbf{x}, \mathbf{x})$ for the S defined above. Is T linear?
8. Describe the possible row-echelon forms for the associated matrix for T where $T : \mathbb{R}^m \to \mathbb{R}^n$ is linear and onto.
9. Let $A = \begin{bmatrix} 2 & 2 & 3 \end{bmatrix}$. Find a basis for $\ker(A)$ and a basis for $\ker(A^T)$.
10. Suppose that A, B are square matrices of the same size and that $col(B) \subseteq \ker A$. What can I say about AB?
11. Let $A = [a_{ij}]$ where $a_{ij} = i + j$ for $1 \le i, j \le n$. Determine $rank(A)$.
12. Suppose that A is $m \times n$. Show that $\ker A = \ker(UA)$ for all invertible $m \times m$ matrices U. Then also show that $\dim(\ker(A)) = \dim(\ker(AB))$ for all invertible $n \times n$ matrices B.