INSTRUCTIONS

Here's how I recommend using this mock final exam.

(i) Find a day where you can set aside 3 hours at the scheduled exam time (i.e. 2:00-5:00 PM).

(ii) Set a timer for 3 hours.

(iii) Go through the exam as you would on the test date. Simulate test conditions as accurately as possible.

(iv) Afterwards, grade yourself (you'll have to estimate reasonable point deductions - **don't grade yourself gently**). Be your own worst critic. If you write something that you think is clear but aren't sure, you should show it to a friend as well to double check.

(v) On places you missed points, use this as an opportunity to review selected material.

(vi) If you found yourself struggling to finish in the allowed time this indicates you're not as solid on the material as you might have thought.

(vii) It might be nice to take it with a friend or a study group and discuss your results with each other at the end.

Now, here's how **NOT** to use this mock final.

(i) **Don't** flip through it before sitting and taking it (i.e. no peeking!) You're really only hurting yourself by doing this.

(ii) **Don't** take it and go easy on yourself either with time or with grading.

(iii) **Don't** view this as any assurance of the difficulty of the actual test. I am aiming for similarity in style and a **rough approximation** of difficulty but, for what I hope are obvious reasons, this isn't ever actually possible.

(iv) **Don't** use your (presumed excellent) performance on this as a reason to become complacent. There's a lot of material from which you can be tested so there's plenty of further opportunity to be challenged on the test day.

FACULTY OF ARTS & SCIENCES
University of Toronto

MAT223: Linear Algebra

PRACTICE Final Exam, Winter 2018

Duration: 180 minutes

Total: 173 points

Family Name: ______________________________

(Please Print – You will lose 3 points for getting this incorrect)

Given Name(s): ______________________________

(Please Print)

UTorId: ______________________________

Toronto Email Address: ______________________________

Student Number: ______________________________

Signature: ______________________________

There are 9 questions in total. The last page(s) of the booklet are blank and are there for scratch paper. DO NOT REMOVE any extra page. If you happen to need to use the last page clearly indicate on whatever question you're working on that it is continued on the extra page.

Do NOT begin until you are instructed to do so.

When you are told the test has ended you MUST stop writing at once. Failure to do so is an academic offence.

Electronic Aids of any kind are Forbidden.

Problem 1. (12 points, 4 points each). Short answer.

(i) Define what it means for a set of vectors to be a basis for a subspace S.

(ii) State the Cauchy-Schwartz inequality.

(iii) Define the rank of a matrix.

Problem 2. (20 points). Consider subspace $U = \{\begin{bmatrix} a-3b+6c \\ 5a+4d \\ b-2c-d \\ 5d \end{bmatrix} | a,b,c,d \in \mathbb{R}\}$.

(i) (10 points). Calculate $\dim(U)$.

(ii) (10 points). Give a basis for the orthogonal complement of U.

Problem 3. (10 points). Suppose that $b \neq 0$. Show that $\mathbf{n} = \begin{bmatrix} a \\ b \end{bmatrix}$ is orthogonal to all vectors parallel to the line $ax + by + c = 0$.

Problem 4. (30 points). Let $\mathbf{u}, \mathbf{v}$ be in $\mathbb{R}^n$ where $\mathbf{u}$ is nonzero. Define $A = \mathbf{u}\mathbf{v}^T$.

(i) (5 points). Calculate $rank(A)$?

(ii) (10 points). Prove that $\mathbf{u}$ is an eigenvector of A. What is its eigenvalue?

(iii) (5 points). What are the other eigenvalues of A? Justify your answer.

(iv) (10 points). Is A diagonalizable? Explain.

Problem 5. (17 points). Determine whether the matrix $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ is diagonalizable and, if so, diagonalize it.

Problem 6. (20 points). Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation and let S be the collection of all points in $\mathbb{R}^n$ which are equal to their image under T. Prove that S is a subspace of $\mathbb{R}^n$.

Problem 7. (15 points total). Let A be an $n \times n$ symmetric matrix. Show that eigenvectors of A corresponding to *different* eigenvalues must be orthogonal.

Problem 8. (10 points). Consider the LU factorization of a matrix A given by $\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Use the LU factorization to solve $A\mathbf{x} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$. *Note: You receive a score of zero if you calculate the matrix A or if you use the reduction algorithm.*

Problem 9. (39 points total, 3 points each) This problem continues on the next page. For the following questions, answer using the word "True" or the word "False". You **don't need to justify your answer** to receive full credit. However, **to discourage guessing we will deduct 2 points for each incorrect answer given**.

(i) If $A\mathbf{x} = 0$ has only the trivial solution then A is invertible.

(ii) Any nonzero element of $Nul(A)$, for a square matrix A, is an eigenvector of A.

(iii) The set $S = \{\begin{bmatrix} x \\ y \end{bmatrix} \mid |x| + y = 0\}$ is a subspace of $\mathbb{R}^2$.

(iv) If A and B are two $n \times n$ matrices then $\det(A + B) = \det(A) + \det(B)$

(v) If the $n \times n$ matrix A has n distinct eigenvalues, it must be diagonalizable.

(vi) Any multiple of an eigenvector of a given matrix is also an eigenvector *with the same eigenvalue*.

(vii) The characteristic polynomial of a matrix must have real roots.

(viii) The characteristic polynomial of a matrix must have nonzero roots.

(ix) Diagonalizable matrices are invertible.

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(x) Let $\mathbf{d} \neq 0$. The projection operator $T(\mathbf{x}) = \mathbf{proj_d}(\mathbf{x})$ is a linear transformation.

(xi) If A is 3×3 then $\det(A^4) = (\det(A))^{12}$.

(xii) If A is $n \times n$ then $\det(I_n \det(I_n \det(A))) = \det(A)$.

(xiii) If $c_A(y^2) = 0$ where $c_A(\lambda)$ is the characteristic polynomial of A then y is an eigenvalue of A.

(*Nothing on this page below here will be graded but is extra space if you want it*)

(*extra paper*)

(*extra paper*)