Instructions: READ THIS

This mock midterm contains questions pertaining to linear independence. By the time this mock midterm has been made public, some of you may not have seen this topic in lecture section. Wait until you've seen it in lecture or read about it on your own before continuing. Here's how I recommend using this mock midterm.

(i) Find a day where you can set aside 50 minutes at the scheduled exam time (i.e. 4:10-5:00 PM).

(ii) Set a timer for 50 minutes.

(iii) Go through the exam as you would on the test date. Simulate test conditions as accurately as possible.

(iv) Afterwards, grade yourself (you'll have to estimate reasonable point deductions - **don't grade yourself gently**).

(v) On places you missed points, use this as an opportunity to review selected material.

(vi) If you found yourself struggling to finish in the allowed time this indicates you're not as solid on the material as you might have thought.

(vii) It might be nice to take it with a friend or a study group and discuss your results with each other at the end.

Now, here's how **NOT** to use this mock midterm.

(i) **Don't** flip through it before sitting and taking it (i.e. no peeking!) You're really only hurting yourself by doing this.

(ii) **Don't** take it and go easy on yourself either with time or with grading.

(iii) **Don't** view this as any assurance of the difficulty of the actual test. I am aiming for similarity in style and a rough approximation of difficulty but, for what I hope are obvious reasons, this isn't ever actually possible.

(iv) **Don't** use your (presumed excellent) performance on this as a reason to become complacent. There's a lot of material from which you can be tested so there's plenty of further opportunity to be challenged on the test day.

FACULTY OF ARTS & SCIENCES
University of Toronto

MAT223: Linear Algebra

PRACTICE Midterm Exam #1

Duration: 50 minutes

Total: 90 marks

Family Name: ______________________________

(Please Print- You will lose 3 points for getting this wrong)

Given Name(s): ______________________________

(Please Print)

Student Number: ______________________________

Toronto Email: ______________________________

Signature: ______________________________

You may NOT use calculators, or any electronic devices during the test. You must completely justify your answers. Do NOT remove any pages from the test booklet.

FOR MARKER'S USE ONLY			
Problem 1:	/20	Problem 2:	/25
Problem 3:	/15	Problem 2:	/15
Problem 5:	/15		
		TOTAL:	/90

Problem 1. (20 points). Use the reduction algorithm to solve (if possible) $A\mathbf{x} = \mathbf{b}$ for

$$A = \begin{bmatrix} 3 & 5 & -4 \\ -3 & -2 & 4 \\ 6 & 1 & -8 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 7 \\ -1 \\ -4 \end{bmatrix}$$

Give your answer (if one exists) as a vector solution.

Problem 2. (25 points 5 points each). Short answers.

(i) List the Elementary Row Operations.

(ii) How many 3×2 matrices in reduced row echelon form are there?

(iii) How many 4×1 matrices in reduced row-echelon form are there?

(iv) Given a nonzero vector $\mathbf{d} \in \mathbb{R}^n$ and $\mathbf{x} \in \mathbb{R}^n$ *define* $\mathbf{proj_d x}$.

(v) Suppose A is a 3×3 matrix and $\mathbf{b} \neq \mathbf{0}$ is a 3×1 column vector. Is the solution set to $A\mathbf{x} = \mathbf{b}$ necessarily a plane through the origin? Explain why or why not.

Problem 3. (15 points) Let $\{\mathbf{v}_1, ..., \mathbf{v}_k\} \subset \mathbb{R}^n$ be nonzero mutually orthogonal vectors (i.e. every vector in the set is orthogonal to every other vector in the set) and define the $n \times k$ matrix $A = \begin{bmatrix} \mathbf{v}_1 & \cdots & \mathbf{v}_k \end{bmatrix}$. Show that the only solution to $A\mathbf{x} = \mathbf{0}$ is the trivial solution.

Problem 4. (15 points) The graph of an equation of the form $ax + by + cz = 0$ is a plane through the origin in $\mathbb{R}^3$. Prove that two planes through the origin must have infinitely many points in common.

Problem 5. (15 points) Let $a_1, ..., a_n$ be positive real numbers. Prove that

$$(a_1 + a_2 + \cdots a_n)(\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n}) \geq n^2$$

(*extra paper*)