Instructions: READ THIS

This mock midterm contains questions pertaining to matrix multiplication (not just matrix-vector multiplication). By the time this midterm has been made public, some of you may not have seen this in lecture section. Wait until you've seen it in lecture or read about it on your own before continuing. Here's how I recommend using this mock midterm.

(i) Find a day where you can set aside 50 minutes at the scheduled exam time (i.e. 4:10-5:00 PM).

(ii) Set a timer for 50 minutes.

(iii) Go through the exam as you would on the test date. Simulate test conditions as accurately as possible.

(iv) Afterwards, grade yourself (you'll have to estimate reasonable point deductions - **don't grade yourself gently**).

(v) On places you missed points, use this as an opportunity to review selected material.

(vi) If you found yourself struggling to finish in the allowed time this indicates you're not as solid on the material as you might have thought.

(vii) It might be nice to take it with a friend or a study group and discuss your results with each other at the end.

Now, here's how **NOT** to use this mock midterm.

(i) **Don't** flip through it before sitting and taking it (i.e. no peeking!) You're really only hurting yourself by doing this.

(ii) **Don't** take it and go easy on yourself either with time or with grading.

(iii) **Don't** view this as any assurance of the difficulty of the actual test. I am aiming for similarity in style and a **rough approximation** of difficulty but, for what I hope are obvious reasons, this isn't ever actually possible.

(iv) **Don't** use your (presumed excellent) performance on this as a reason to become complacent. There's a lot of material from which you can be tested so there's plenty of further opportunity to be challenged on the test day.

FACULTY OF ARTS & SCIENCES
University of Toronto

MAT223: Linear Algebra

PRACTICE Midterm Exam #2

Duration: 50 minutes

Total: 100 marks

Family Name: ______________________________
(Please Print)

Given Name(s): ______________________________
(Please Print)

Student Number: ______________________________

Signature: ______________________________

You may NOT use calculators, or any electronic devices during the test. You must completely justify your answers. Do NOT remove any pages from the test booklet.

FOR MARKER'S USE ONLY			
Problem 1:	/20	Problem 2:	/15
Problem 3:	/20	Problem 4:	/15
Problem 5:	/30		
		TOTAL:	/100

Problem 1. (20 points). Solve the system

$$\begin{aligned} 3x - y &= 5 \\ 2x + 2y &= 1 \end{aligned}$$

by using the inverse of the associated coefficient matrix. (*Note: No other method of solution will be graded.*)

Problem 2. (15 points total). Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Write A as a product of elementary matrices.

Problem 3. (20 points). Suppose that A is an $n \times n$ matrix. Prove that if $\mathbf{x} \in \text{Nul}A$ and $\mathbf{b} \in col(A^T)$ then $\mathbf{x}$ and $\mathbf{b}$ are orthogonal. (Hint: It may help to recall that $[\mathbf{x} \cdot \mathbf{y}] = \mathbf{x}^T \mathbf{y}$.)

Problem 4. (15 points total). Suppose that $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a transformation defined by

$$T(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}) = \begin{bmatrix} x_1 + x_2 \\ x_1 - x_2 \\ x_1 + x_2 \end{bmatrix}$$

(i) (10 points). Is T linear? If yes, what is the standard matrix for T? If no, explain why not.

(ii) (5 points). Is T invertible? (*Note: You can answer this without knowing the answer to part (i) if you don't have a lot of confidence in your answer to part (i)*).

Problem 5. (30 points total, 5 points each) For the following questions, answer using the word “True” or the word “False”. You don’t need to justify your answers.

(i) If an $n \times n$ matrix A can be written as a product of elementary matrices then $col(A) = \mathbb{R}^n$

(ii) If $A\mathbf{x} = \mathbf{0}$ has only the trivial solution then A is invertible.

(iii) If a transformation T from $\mathbb{R}^n \to \mathbb{R}^m$ is linear, then it is represented by a matrix, $T(\mathbf{x}) = A\mathbf{x}$ for A an $n \times m$ matrix.

(iv) If A is an invertible matrix then so is AA^T.

(v) If A is an invertible then so is cA for all $c \in \mathbb{R}$.

(vi) If A and B are both invertible $n \times n$ matrices then $A + B$ is invertible as well.

(*Problem 3 extra paper*)