Topics Covered so Far

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Below is a list of topics considered "fair game" for the upcoming midterm on March 9. A few of these you may have not seen yet **but you will by the test date**. In a perfect world (for me, not you) I'd have enough time to test you on all of these but for obvious lack of time I have to select concepts I want to focus on. The list below just recaps a lot of what we've covered to help jog your memory and help you study.

1. **Systems of linear equations:** What they are, related vocabulary, geometrical interpretation. Solutions and consistency, *existence/uniqueness* etc. You should also be able to give solutions in a parametric vector form as in, for example, on page 45.

2. **Augmented matrices:** What they are, how to manipulate them via **elementary row operations**. This gave rise to the notion of **equivalence** between systems of linear equations.

3. **Reduction Algorithm:** using the reduction algorithm to produce the **general solution** to a given linear system. This meant learning about **row-echelon** and **reduced row-echelon** form for a matrix.

4. **Pivots:** We learned about pivot position, pivot columns, and pivots. You need to know what they are and how they were used. As well you need to know about **basic variables** and **free variables** and how they are used (as in building **parametric descriptions** of solution sets.

5. **Vectors:** You need to know about vectors in $\mathbb{R}^n$ i.e. what they are and the algabra of manipulating them (as in the table on page 27).

6. **Vector Geometry:** You'll need to know about the **dot product** in $\mathbb{R}^n$ and related ideas like **length** of a vector and about the *angle between two vectors* in $\mathbb{R}^2$ as implicitly determined by the **dot product**. You also need to know about **orthogonality** and related concepts like the **Cauchy** and **Triangle** inequalities.

7. **Matrix-vector product:** If given an $m \times n$ matrix A and an $n \times 1$ vector $\mathbf{x}$ you should know how to compute $A\mathbf{x}$ – both in terms of taking a combination of the columns of A as well as using the dot product of the rows of A with $\mathbf{x}$. You'll need to know when the columns of A span $\mathbb{R}^m$ and how $A\mathbf{x} = \mathbf{b}$ is solved (if solvable).

8. **Homogeneous Systems:** You should know these are consistent. You should know words like **trivial solution** and **nontrivial solution** etc. You should know when they are guaranteed to have infinitely many solutions.

9. **Linear Combinations:** This is a fundamental recurring theme in this course and you need to know what it means.

10. **Spanning:** Given a collection of vectors you need to know what is meant by the **span** of the collection of vectors.

Below is material covered since the first test

11. **Linear Independence:** You need to know what's meant by linear dependence and independence and how to tell whether a given collection of vectors is or is not linearly dependent/independent. Independence is, at heart, **a geometric concept**.

12. **Transformations:** You need to know what a transformation is and how to know *whether a given transformation is linear*, namely how this concept relates to the idea of linear combinations. We've seen that linear transformations are induced by associated matrix multiplications and vice versa. This is summarized in Theorem 10 on page 72 which you need to fully understand. You should be able to write down the **standard matrix** for a given linear transformation. Composition of linear transformations preserves the linearity and therefore results in a new linear transformation. Moreover, composition of linear transformations corresponds to multiplication of the associated matrices from which they are respectively induced. I.e., provided the sizes of A, B are compatible $T_A \circ T_B = T_{AB}$.

13. **Transformation jargon:** You need to know all the right terminology like "domain", "codomain", "onto" etc. You should be able to know how to answer questions about various properties of a transformation like determining if it's linear or whether it's onto etc.

14. **Matrix Arithmetic:** You need to know how to do scalar multiplication, addition, and the various arithmetical manipulations on matrices based on these. You need to know about size, and equality, and entries (*how to specify matrices given a description of the entries using indices*). You should know special matrices like the **zero matrix** and the **identity matrices**.

15. **Matrix Multiplication:** You need to know how to multiply matrices. This includes matrix-vector products, but also the more general case of compatible matrices. You should know the **dot product rule** for quickly multiplying matrices with compatible sizes. You should be able to look at matrices and tell if they're compatible. You should know the algebraic rules of matrices (associativity, lack of commutativity, etc) with respect to multiplication.

16. **Transposes:** You need to know about this, and associated concepts (like **symmetric matrices**). You should know how multiplication and the transpose operation interact with each other.

17. **Inverses:** You should know about matrix inverses, like the (incomplete list of) equivalent conditions on a matrix being invertible given on page 114. You should know how to find the inverse of an invertible matrix. As well, you should know how to use the **determinant** to find the inverse (and check invertibility) of a 2×2 matrix.

18. **Elementary Matrices:** You need to what they are, how to tell if a matrix is elementary, how to produce your own elementary matrices given a description of associated row operations, how to find the inverse of an elementary transformation etc.

19. **LU Factorization:** You should know about the LU decomposition and how it's used to solve systems of linear equations, when do matrices have LU decompositions, etc.

20. **Spanning:** Given a collection of vectors you need to know what is meant by the span of the collection of vectors.

21. **Subspaces:** You need to know how to tell if a given subset is a subspace. There are canonical subspaces associated to a given matrix A called **null space** and **column space**, denoted null A and col$A$ respectively.

General Things

1. **Grammar, grammar, grammar:** Words have *very rigid usages* in mathematics. Your articulation of some concept or rationale behind a calculation has to be worded *correctly* and *clearly*. Here's are examples of nonsensical (and therefore **incorrect**) statements:

   (a) "The vector inverse does not exist."

   (b) "The matrix spans $\mathbb{R}^m$"

   (c) "The matrix is linearly independent...."

   You should be able to point to what's wrong in each of the above statements. If you write similarly ambiguous or meaningless things on a test you shouldn't expect to get points. If this were an English class, the above statements would be as meaningless as the sentence "The car's neck stuck". We only know what you think *based on what you write* so if you do not articulate your thoughts completely clearly and accurately we have to assume your thoughts are similarly unclear or inaccurate.

2. **Logical flow:** The presentation of a formal argument should be structured in a clear, precise way starting from the premises and concluding with the desired result. Often proofs in this course require very little more than a solid grasp of definitions. Some require more cleverness or insight but in general beginning with the definitions is a good way to at least get started. Be sure that in moving from one step of a proof to another you are not introducing new (maybe subtle) assumptions into your argument.