Juniors
(Grades up to 10)

International Mathematics
TOURNAMENT OF THE TOWNS

A-Level Paper **Spring 2006.**[1]

1 **[4]** There is a billiard table in shape of rectangle 2×1, with pockets at its corners and at midpoints of its two largest sizes. Find the minimal number of balls one has to place on the table interior so that any pocket is on a straight line with some two balls. (Assume that pockets and balls are points).

2 **[4]** Prove that one can find 100 distinct pairs of integers such that every digit of each number is no less than 6 and the product of the numbers in each pair is also a number with all its digits being no less than 6.

3 **[5]** On sides AB and BC of an acute triangle ABC two congruent rectangles $ABMN$ and $LBCK$ are constructed (outside of the triangle), so that $AB = LB$. Prove that straight lines AL, CM and NK intersect at the same point.

4 **[5]** Is there exist some positive integer n, such that the first decimal of 2^n (from left to the right) is 5 while the first decimal of 5^n is 2?

5 **[6]** Numbers 0, 1 and 2 are placed in a table 2005×2006 so that total sums of the numbers in each row and in each column are factors of 3. Find the maximal possible number of 1-s that can be placed in the table.

6 **[7]** Let us call a pentagon curved, if all its sides are arcs of some circles. Are there exist a curved pentagon P and a point A on its boundary so that any straight line passing through A divides perimeter of P into two parts of the same length?

7 Anna and Boris have the same copy of 5×5 table filled with 25 distinct numbers. After choosing the maximal number in the table, Anna erases the row and the column that contain this number. Then she continue the same operations with a smaller table till it is possible. Boris basically does the same; however, each time choosing the minimal number in a table. Can it happen that the total sum of the numbers chosen by Boris

a) **[6]** is greater than the total sum of the numbers chosen by Anna?

b) **[2]** is greater than the total sum of any 5 numbers of initial table given that no two of the numbers are in the same row or in the same column?

[1] Your total score is based on the three problems for which you earn the most points. Points for each problem are shown in brackets [ ].