UNIVERSITY OF TORONTO
DEPARTMENT OF MATHEMATICS
MAT 235 Y - CALCULUS FOR PHYSICAL AND LIFE SCIENCES II
FALL-WINTER 2002-2003
GENERAL INFORMATION ABOUT TEST #3.

1. DATE / TIME.
   Tuesday, March 4, from **6:00 to 8:00 p.m.**
   Students with **timetable conflicts** will write the test on the **same day** from **4:00 p.m. to 6:00 p.m.**
2. LOCATIONS.
   Section **L-0101** (Prof. **Abou-Ward)** writes the test in room **CG-150** (**Canadiana Gallery)**
   Section **L-0201** (Prof. **Uppal)** writes the test in room **CG-250** (**Canadiana Gallery)**
   Section **L-5101** (Prof. **Recio**) writes the test in room **WW-111** (**Woodsworth College**)
   The **4:00 p.m. to 6:00 p.m.** test, for students with **timetable conflicts** from any of the above sections, will be written in room **MS-3153** (**Medical Sciences Building**).
3. ABOUT THE TEST.
   Topics to be covered: **textbook chapters 16 (sections 16.5, 16.6, 16.7, 16.8 , 16.9) and 17 (sections 17.1, 17.2, 17.3, 17.4, 17.5)**.
   Duration: **2 hours**. Value: **20% of course mark**. Aids allowed: **calculators or any other aids are not allowed**.
4. MATH AID CENTRES.
   Sidney Smith Math Aid Centre. Location: Sidney Smith Building, room SS-1071
   Hours of operation: Posted outside room SS-1071
   Note: A tutor for MAT 235 is available at this location every Wednesday from 12 noon to 2 p.m. and every Thursday from 4 p.m. to 6 p.m.
   Victoria College Math Aid Centre. Location: Victoria College Building, room 006.
   Hours of operation: Monday to Thursday, from 12:00 noon to 3:00 p.m.
   St. Michael's Math Aid Centre. Location: John M. Kelly Library, room 202.
   Hours of operation: Monday to Thursday, from 12:00 noon to 3:00 p.m.
   Woodsworth College Math Aid Centre. Location: Woodsworth College Building, room 115
   Hours of operation: Posted outside room WW-115
   University College Math Aid Centre. Location: University College Building, room 048 (basement)
   Hours of operation: Posted outside room UC-048
   New College Math Aid Centre. Location: New College Building, (basement)
   Hours of operation: Posted outside MAC room.
5. SAMPLE QUESTIONS FROM PREVIOUS TEST #3 PAPERS.

1. a) (10 marks) Compute the surface area of the part of the cone $z^2 = 4(x^2 + y^2)$ that lies between the planes $z = 2$ and $z = 4$.
   b) (15 marks) Evaluate $\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} \int_{0}^{\sqrt{1-x^2-y^2}} (x^2 + y^2 + z^2)^2 \, dz\, dy\, dx$.
2. a) (10 marks) Evaluate the line integral $\int_C z\, ds$, where C is the curve given by the parametrization $x = 2t$, $y = t^3/3$, $z = 2t^3/3$, $0 \le t \le 1$.
   b) (10 marks) Use Green's Theorem to evaluate the line integral $\int_C (y^2 + \sin(x^2))\, dx + (x + \cos(y^2))\, dy$, where C is the triangular curve consisting of the line segments from $(0,0)$ to $(1,0)$, from $(1,0)$ to $(1,1)$, and from $(1,1)$ to $(0,0)$.

3. a) (10 marks) Let $\mathbf{F}(x, y, z) = x\,\mathbf{i} + z\,\mathbf{j} + a\,y\,\mathbf{k}$. Find all the values of the constant a, if any, for which $\operatorname{div}(\mathbf{F} \times \operatorname{curl} \mathbf{F}) = \operatorname{div} \mathbf{F}$.

b) (15 marks) Let $\mathbf{G}(x, y) = e^{-2y} \sin x\,\mathbf{i} + (2e^{2y} + 2e^{-2y} \cos x)\,\mathbf{j}$. Find a function $g(x, y)$ such that $\nabla g = \mathbf{G}$, and use it to evaluate the line integral $\int_C \mathbf{G} \cdot d\mathbf{r}$, where C is the arc of the curve $y = \cos^3 x$, from $x = 0$ to $x = \pi$.

4. (15 marks) Compute the mass of the solid in the first octant, bounded by the cylinder $y^2 + z^2 = 1$, and the planes $x = 0$, $y = 0$, $z = 0$ and $x + y = 2$, if the density function is $\delta(x, y, z) = 2z/(1 + y)$.

5. (15 marks) Evaluate $\iint_R (x + y)^2\, dA$, where R is the region bounded by the curves $x + y = 2$, $x + y = 4$, $x^2 - y^2 = 4$ and $y = x$. (Hint: Use an appropriate change of variables.)

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1. (15 marks) Find the area of the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 2$.

2. (15 marks) A lamina occupies the region $D = \{(x, y) \mid -\pi/2 \le x \le \pi/2,\ 0 \le y \le \cos x\}$ and has density function $\rho(x, y) = y$. Find the coordinates of the centre of mass of this lamina.

3. (15 marks) Evaluate $\iiint_R z\, dV$, where R is the solid region in the first octant that lies between the sphere $x^2 + y^2 + z^2 = 1$ and the cone $z = \sqrt{x^2 + y^2}$.

4. (15 marks) Use the transformation $u = x + y$, $v = x - y$ to evaluate the integral $\iint_T (x + y)^{-2}\, dA$, where T is the trapezoidal region with vertices $(1, 1)$, $(3, 3)$, $(6, 0)$, and $(2, 0)$.

5. (10 marks) Let C be the curve of intersection of the surfaces $z = x^2 + y^2$ and $x = 2y$. Determine the work done by the force $\mathbf{F}(x, y, z) = (x - z)\,\mathbf{i} + (1 - z)\,\mathbf{j} + y\,\mathbf{k}$ on a particle that moves along the curve C from $(0, 0, 0)$ to $(2, 1, 5)$.

6. (10 marks) Show that the line integral $\int_C (y^2 \cos(xy)\, dx + (\sin(xy) + xy \cos(xy))\, dy$ is independent of path and evaluate it over any path from $(\pi, 1/2)$ to $(\pi/2, 3)$.

7. (10 marks) Use Green's Theorem to evaluate the line integral $\int_C (e^x - xy)\, dx + (x^2 + \ln(1 + y))\, dy$, where C is the triangle with vertices $(0, 0)$, $(1, 2)$, and $(0, 3)$, positively oriented.

8. a) (5 marks) Let $\mathbf{F}(x, y, z) = (a y^3 - z^2)\,\mathbf{i} + (b z + x y^2)\,\mathbf{j} + (c x z + 3y)\,\mathbf{k}$, where a, b, and c are constants. Compute the curl of $\mathbf{F}$ and find values of a, b, and c, if any, for which $\mathbf{F}$ is conservative.

b) (5 marks) Is there a vector field $\mathbf{G}$ on R^3 such that $\operatorname{curl} \mathbf{G} = x^3\,\mathbf{i} + (z - 2x^2 y)\,\mathbf{j} + (2 + z - x^2 z)\,\mathbf{k}$
?
Justify your answer.