4. a) (5 marks) Consider the function $f(x,y)=\dfrac{x^4+y^2}{x^2+3y^2}$. Does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist?

If the limit exists, then evaluate it. If the limit does not exist, then give reasons to explain why.

b) (5 marks) Consider the function $g(x,y)=\sqrt[3]{x^8+8y^3}$. Determine which of the following is true:

$\dfrac{\partial g}{\partial y}(0,0)=0$, $\dfrac{\partial g}{\partial y}(0,0)=1$ or $\dfrac{\partial g}{\partial y}(0,0)=2$. Give reasons to explain your answer.

a) the limit does not exit. ← 2 marks

3 marks

Take $x=0$, $y=t\neq 0$, then $f(x,y)=\frac{1}{3} \xrightarrow[t\to 0]{} \frac{1}{3}$

Take now $x=t\neq 0$, $y=0$, then $f(x,y)=t^2 \xrightarrow[t\to 0]{} 0$

b) $\dfrac{\partial g}{\partial y}(0,0)=2$ ← 2 marks

For any value of y, $g(0,y)=2y=f(y)$

$\dfrac{\partial g}{\partial y}(0,y)=f'(y)=2$, and $\dfrac{\partial g}{\partial y}(0,0)=2$.

3 marks

5. a) (5 marks) Let $z = x^4 \ln(5 - x^2 y)$. Evaluate $\frac{\partial z}{\partial x}(-1,3)$.

b) (10 marks) Given the function $f(x, y, z) = y\, e^{kx} \sin(3z)$, find all values of the constant k, if any, for which the function f satisfies the differential equation $2 f_{xx} + 3 f_{yy} + 8 f_{zz} = 0$.

a) $\frac{\partial z}{\partial x} = 4x^3 \ln(5 - x^2 y) - \frac{2x^5 y}{5 - x^2 y}$ ← 4 marks

$\frac{\partial z}{\partial x}(-1,3) = -4 \ln 2 + \frac{6}{2} = \boxed{3 - 4\ln 2}$ ← 1 mark

b) $f_x = k y e^{kx} \sin(3z)$ $\qquad f_{xx} = k^2 y e^{kx} \sin(3z)$ ← 2 marks

$f_y = e^{kx} \sin(3z)$ $\qquad f_{yy} = 0$ ← 2 marks

$f_z = 3y e^{kx} \cos(3z)$ $\qquad f_{zz} = -9 y e^{kx} \sin(3z)$ ← 2 marks

For $2 f_{xx} + 3 f_{yy} + 8 f_{zz} = 0$:

$(2k^2 - 72)\, y e^{kx} \sin(3z) = 0$ ← 2 marks

Then: $2k^2 - 72 = 0$ $\qquad \boxed{k = \pm 6}$ ← 2 marks