

UNIVERSITY OF TORONTO
DEPARTMENT OF MATHEMATICS
MAT 235 Y - TEST #2

JULY 20, 1995

NAME:

STUDENT NO.:

(Family name. Please PRINT.) (Given name.)

INSTRUCTIONS: Show and explain all your work in all questions. Give your answers in the space provided. Use both sides of paper, if necessary. Do not tear out any pages. No calculators or other aids are permitted. Time allowed: 2 hours.

1. (15 marks) Determine the extreme values of $f(x,y)=x^2-xy-4x+3y+2$ on the triangular region $0 \leq x \leq 4$, $0 \leq y \leq 2x$.

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2. Evaluate each of the following integrals:

a) (5 marks) $\int_0^2 \int_x^2 x\sqrt{1+y^3} dy dx$ b) (5 marks) $\int_1^2 \int_0^{\infty} \frac{1}{x^2(1+y^2)} dy dx$

c) (10 marks) $\int_0^2 \int_y^{\sqrt{8-y^2}} \frac{y}{1+(x^2+y^2)^{\frac{3}{2}}} dx dy$

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3. (10 marks) Find the area of the surface given by the equation $z=x^2-2\ln x+y\sqrt{15}$ lying over the square $0 \leq y \leq 1$, $1 \leq x \leq 2$.

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4. (15 marks) Find the volume of the region in the first octant bounded above by the surface $z=1-y^2$ and lying between the planes $x=0$ and $x+z=3$.

(15 marks) Evaluate $\iiint_R (x^2 + 3xyz) \, dx \, dy \, dz$ over the region $1 \leq x \leq 2$, $0 \leq xy \leq 2$, $0 \leq z \leq 1$, using the substitution $u=x$, $v=xy$, and $w=3x$.

Evaluate $\int_C \mathbf{F} \cdot d\mathbf{x}$ in each of the following cases:

- a) (5 marks) $F(x,y) = (1+xy, y^2)$, and C is the arc of the circle $x^2 + y^2 = 4$, from $(2,0)$ to $(0,2)$.
- b) (5 marks) $F(x,y,z) = (y^2, 2xy - z^2, 3 - 2yz)$, and C is the arc parametrized by $\mathbf{r}(t) = (\sin(t^3 \pi/2), \cos(t^5 \pi), 2 - t^4)$, $0 \leq t \leq 1$.

7. (15 marks) Given the integral $\int_C (x-3y) \, dx + (x^2+y) \, dy$, where C is the boundary of the region enclosed between the graphs of $y=x^2$ and $y=2x$, verify Green's Theorem by evaluating both, the line integral and the double integral.