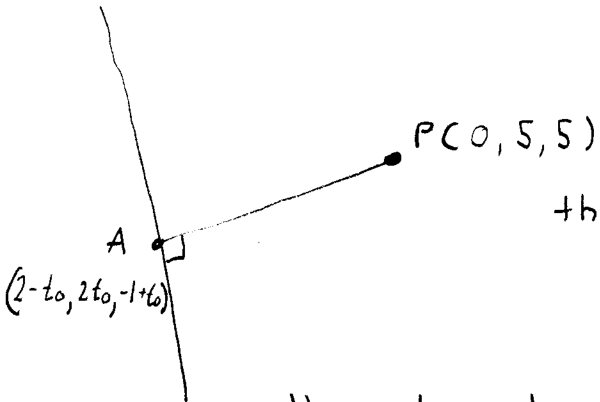
1) a) $x = 2 - t \quad y = 2t \quad z = -1 + t$

so $\vec{v} = (-1, 2, 1)$



$\overrightarrow{AP} = (-2 + t_0, 5 - 2t_0, 6 - t_0)$

know $\vec{v} \cdot \overrightarrow{AP} = 0$

thus $(-1, 2, 1) \cdot (-2 + t_0, 5 - 2t_0, 6 - t_0) = 0$

$2 - t_0 + 10 - 4t_0 + 6 - t_0 = 0$

$-6t_0 = -18$

$t_0 = 3$

the closest pt is $(2 - t_0, 2t_0, -1 + t_0)$

but $t_0 = 3$ so $\boxed{A = (-1, 6, 2)}$

b) since the planes intersect, find to points that satisfy

$x + 2y = 0$

$y - 2z = 1$

choose $A = (2, -1, -1)$ $\quad B = (0, 0, \frac{1}{2})$

taking $P = (1, 0, 0)$ and finding $\overrightarrow{PA}$ and $\overrightarrow{PB}$

$\overrightarrow{PA} = (1, -1, -1)$

$\overrightarrow{PB} = (-1, 0, -\frac{1}{2})$

the normal of the plane is $\overrightarrow{PA} \times \overrightarrow{PB}$

so $\vec{n} = (\frac{1}{2}, \frac{3}{2}, -1)$

thus an equation of the plane is

$$\boxed{x + 3y - 2z = 1}$$

2) a) i) $\vec{u} \neq 0 \quad \vec{v} \neq 0$

$$|\vec{u} \times \vec{v}| = \vec{u} \cdot \vec{v}$$
$$|\vec{u}||\vec{v}|\sin\theta = |\vec{u}||\vec{v}|\cos\theta$$
$$\frac{\sin\theta}{\cos\theta} = 1$$
$$\tan\theta = 1$$
$$\boxed{\theta = \pi/4}$$

ii) $\dfrac{\vec{u} \cdot (\vec{u} \times \vec{v} - \vec{v})}{\|\vec{u} \times (\vec{u} + \vec{v}\|}$ solve using dot product and cross product properties

first take $\vec{u} \cdot (\vec{u} \times \vec{v} - \vec{v})$

$= \vec{u} \cdot (\vec{u} \times \vec{v}) - (\vec{u} \cdot \vec{v})$

but $\vec{u} \cdot (\vec{u} \times \vec{v}) = 0$ so $\vec{u} \cdot (\vec{u} \times \vec{v} - \vec{v}) = -(\vec{u} \cdot \vec{v})$

second take $\|\vec{u} \times (\vec{u} + \vec{v})\|$

$= \|\vec{u} \times \vec{u} + \vec{u} \times \vec{v}\|$

but $\vec{u} \times \vec{u} = 0$ so $\|\vec{u} \times (\vec{u} + \vec{v})\| = \|\vec{u} \times \vec{v}\|$

thus $\dfrac{\vec{u} \cdot (\vec{u} \times \vec{v} - \vec{v})}{\|\vec{u} \times (\vec{u} + \vec{v}\|} = \dfrac{-(\vec{u} \cdot \vec{v})}{\|\vec{u} \times \vec{v}\|} = -1$

b) $\vec{u} = (1, 0, -1) \quad \vec{v} = (1, 2, a)$

$|\vec{u} \times \vec{v}| = (-2, -a-1, 2) = \sqrt{4 + a^2 + 2a + 1 + 4}$

$= \sqrt{a^2 + 2a + 9}$

$\vec{u} \cdot \vec{v} = (1, 0, 1) \cdot (1, 2, a)$

$= 1 - a$

so $|\vec{u} \times \vec{v}| = \vec{u} \cdot \vec{v}$

$$\sqrt{a^2 + 2a + 9} = 1 - a$$
$$a^2 + 2a + 9 = 1 - 2a$$
$$4a = -8$$
$$\boxed{a = -2}$$

3) a) $x = 3(t^2-2)$ $y = t(3-t^2)$

Find $\frac{dy}{dx}$, so $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

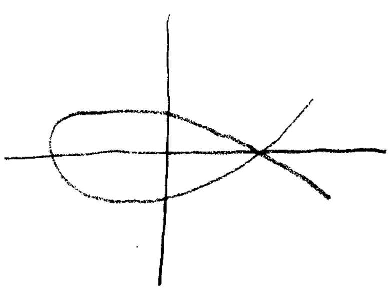
$= \frac{3-3t^2}{6t} = \boxed{\frac{1-t^2}{2t}}$

Find $\frac{d^2y}{dx^2}$, so $\frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{-(t^2+1)}{2t^2}}{6t}$

thus at $t = \frac{1}{2}$ $\frac{d^2y}{dx^2} = \frac{\frac{-(\frac{1}{4}+1)}{2(1/4)}}{3} = \frac{-\frac{5/4}{1/2}}{3} = \frac{-10/4}{3} = \frac{-10}{12} = \boxed{\frac{-5}{6}}$

b) $y = t(3-t^2)$; set $y = 0$ then $0 = t(3-t^2)$

$t = 0$ or $t = \pm\sqrt{3}$



the length of the loop from 0 to $\sqrt{3}$ multiplied twice

$L = 2\int_0^{\sqrt{3}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$= 2\int_0^{\sqrt{3}} \sqrt{(6t)^2 + (3-3t^2)^2}\, dt$

$= 2\int_0^{\sqrt{3}} \sqrt{(3t^2+3)^2}\, dt$

$= 2\int_0^{\sqrt{3}} 3t^2+3\, dt$

$= 2\left[t^3 + 3t \Big|_0^{\sqrt{3}} \right]$

$= 2[3\sqrt{3} + 3\sqrt{3}]$

$= 2[6\sqrt{3}]$

$\boxed{= 12\sqrt{3}}$