

DEPARTMENT OF MATHEMATICS  
UNIVERSITY OF TORONTO

MAT 235Y - CALCULUS II  
Tuesday November 14, 2000

Student Number:

Last Name:

Given Name:

Answer all questions in the space provided.

No aids are allowed.

FOR MARKER ONLY	
Question	Mark
1	/10
2	/20
3	/30
4	/15
5	/15
6	/10
TOTAL	/100

1. [110 marks]

(a) Describe the surface with equation  $x^2 + 4y^2 - z^2 = 1$  and sketch its graph.

(b) Describe the surface  $x^2 - 2x + 4y^2 - z^2 = 0$ .

2. [20 marks]

At the point where  $t = 1$  on the curve with vector equation  $\mathbf{r}(t) = (t, \frac{t^2}{2}, 3t)$  find the unit tangent vector, the unit normal vector, the curvature and the osculating plane.

3. [30 marks]

(a) Find parametric equations for the line  $L$  passing through the points  $Q = (1, 0, 5)$  and  $R = (3, 2, 1)$ .

(b) Find an equation for the plane  $\Pi$  which is perpendicular to  $L$  and contains the point  $P = (2, 1, -3)$ .

3. (c) Find the point of intersection of  $\Pi$  and  $L$ .

(d) Find the distance from  $P$  to the line  $L$ .

3. (e) Find the area of the triangle PQR .

(f) Find the equation of the plane containing L and P.

4. [15 marks

Given the parametric curve  $x = t^3 - t$ ,  $y = t^2 + 1$  find  $dy/dx$  and  $d^2y/dx^2$ . Also find the area between the curve and the x-axis in the parameter interval  $0 < t < 1$ .

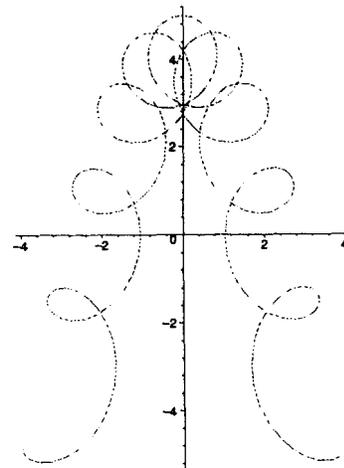
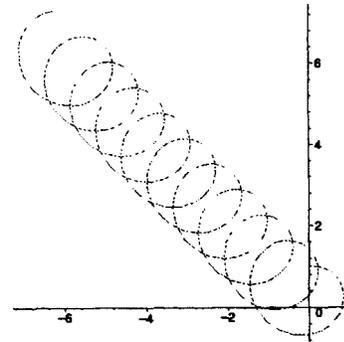
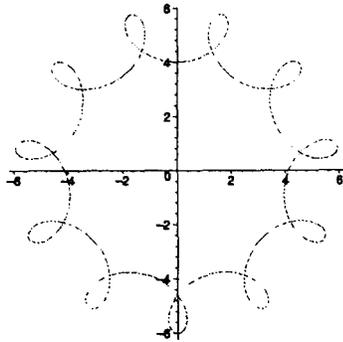
5. [15 marks]

Sketch on the same axes the two curves whose equations in polar co-ordinates are  $r = 1 + \cos(\theta)$  and  $r = 1 + \sin(\theta)$ . Find the area bounded by these two curves in the third quadrant.

6. [10 marks]

Match each of the following parametric plots with its equations by writing its Roman numeral (I, II, III, IV or V) in the box next to the appropriate set of equations.

- |     |  |        |
|-----|--|--------|
| (a) | $x = -t + \sin(10t), y = -t^2 + 4 + \cos(10t)$       | [    ] |
| (b) | $x = \cos(5t) \cos(t), y = \sin(5t) \sin(t)$         | [    ] |
| (c) | $x = 5 \cos t + \sin(10t), y = 5 \sin t + \cos(10t)$ | [    ] |
| (d) | $x = (t + 1)t, y = (t - 1)(t + 2)(t - 2)$            | [    ] |
| (e) | $x = -t + \sin(10t), y = t + \cos(10t)$              | [    ] |



N.