

UNIVERSITY OF TORONTO  
DEPARTMENT OF MATHEMATICS

MAT 235 Y - CALCULUS FOR PHYSICAL AND LIFE SCIENCES  
TEST #3. MARCH 12, 1997

NAME:

\_\_\_\_\_ (Family name. Please PRINT.)

STUDENT No.:

\_\_\_\_\_ (Given name.)

INSTRUCTIONS: This test consists of eight questions. Each question has a value of 10 marks. Total marks: 80. Show all your work in all questions. Give your answers in the space provided. Use both sides of the paper, if necessary. Do not tear out any pages. No calculators or any other aids are permitted. This test is worth 20% of your course grade. Time allowed: 2 hours.

1. Express  $\int_0^1 \int_0^{\sqrt{4-y^2}} f(x,y) dx dy$  as an iterated integral (or integrals) in the opposite order.

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2. Evaluate  $\iint_R 12xy^2 dA$ . R is the region inside  $x^2 + y^2 = 4$  and above the line  $x - y + 2 = 0$ .

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3. Compute the volume of the region between the xy-plane and the graph of the function  $f(x,y) = \sin(\pi \sqrt{x^2 + y^2})$  over the region  $\{(x,y): x^2 + y^2 \leq 1, xy \geq 0\}$ .

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4. Find the area of the part of the sphere  $x^2 + y^2 + z^2 = 1$  that lies over the region inside the circle  $r = \cos\theta$ .

5. Determine the coordinates of the centroid of the region  $x^2 + y^2 + z^2 \leq 1$ ,  $x \geq 0$ ,  $y \geq 0$ , assuming constant density.

6. Evaluate  $\iiint_R x^2 dV$ , where  $R$  is the region  $4x^2 + 9y^2 \leq z + 3 \leq 36$ , using the substitution  $u = 2x$ ,  $v = 3y$ ,  $w = z + 3$ .

7. Evaluate  $\int_C \mathbf{F} \cdot d\mathbf{r}$ , where  $\mathbf{F}(x, y, z) = (x + y, z - 2, z)$  and  $C$  is the semicircle  $y^2 + z^2 = 4$ ,  $z \leq 0$ ,  $x = 2$ .

8. Determine whether the line integral  $\int_{(-2,3,1)}^{(0,2,1)} (2x + yze^{xz}) dx + e^{xz} dy + xye^{xz} dz$  is path independent. Evaluate the integral if it is path independent.